The Role of International Reserves in Sovereign Debt Restructuring under Fiscal Adjustment*

Tiago Tavares†
Department of Economics, ITAM-CIE

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Abstract

Highly indebted developing economies commonly also hold large external reserves. This behavior seems puzzling given that governments borrow with an interest rate penalty to compensate lenders for default risk. Although reducing external debt to the same extent as international reserves would reduce the interest payment burden, reserves can have additional insurance benefits during default crises. Moreover, reserves can also be used to improve lenders recovery rates upon default, thus decreasing the interest rate penalty in non-defaulting times. A standard model of sovereign default risk, augmented with distortionary tax policies and debt restructuring, can replicate quantitatively the observed data patterns on external debt and reserves holdings.

JEL classification: F32, F34, F41, E62

Keywords: Sovereign default, international reserves, distortionary taxation, external debt, sudden stops, debt renegotiation

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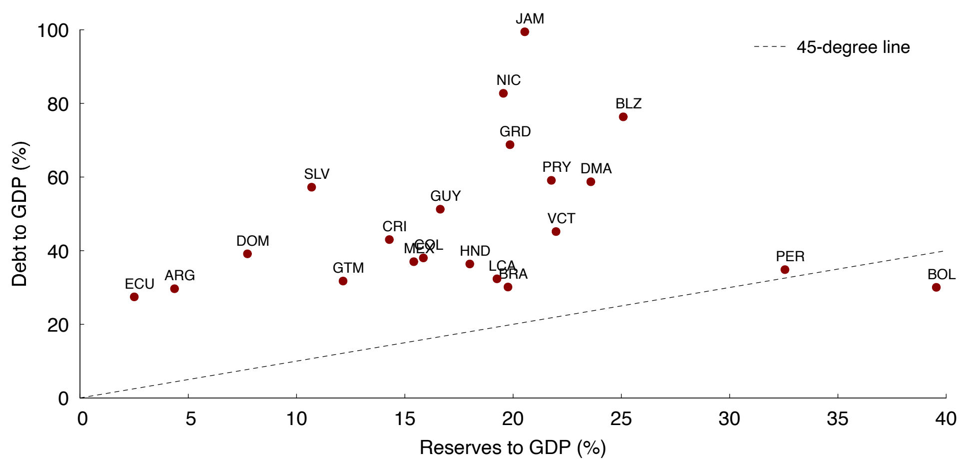
†Instituto Tecnológico Autónomo de México (ITAM), Centro de Investigación Económica (CIE). Camino a Santa Teresa 930, Colonia Heroes de Padierna, Magdalena Contreras CDMX 10700, Mexico. Email: tiago.gomes@itam.mx.

1 Introduction

International reserves are important policy tools in developing economies. A clear pattern from the data shows that many countries decide to hold large positions of international reserves while, at the same time, maintaining high levels of external debts. This observation has intrigued economists that raised questions about the optimal management of international reserves. Holding reserves while keeping positive debt levels entails a cost when the borrowing interest rate is larger than the interest rate earned on liquid foreign assets. For instance, by the end of 2015, governments of emerging economies held on average more that 22% of GDP in international reserves, while external debts were on excess of 52% of GDP.[1] This behavior, not exclusive to a few Asian countries, seems to be more general and also includes Latin American countries (figure 1). Departing from this observation, Rodrik (2006) estimated that these countries can incur in an average annual GDP loss of 1% for maintaining a choice of high debt and reserves. Others have, however, argued that this cost is outweighed by the benefit that reserves provide as an insurance instrument against the occurrence of financial crises (Feldstein, 1999). This paper deals with the question of why developing economies hold simultaneously large amounts of debt and reserves, and what kind of financial crises are more prone to induce such choice.

Figure 1: International reserves and external debt for Latin American countries in 2015



Source: *World Development Indicators*
Notes: Only Latin American countries that fall in the category *Middle Income* from the World Bank are included in the figure. The three letters on top of each dot correspond to the 3 digit country code used in the United Nations. All values of external debt and international reserves refer to the year of 2015.

To analyze the above question, an Eaton and Gersovitz (1981) sovereign default model is developed with four new features. First, reserves accumulation is explicitly modeled by allowing the

[1]These figures use data from the *World Development Indicators* of the World Bank restricting the sample using its classification of *Middle Income* countries with a population of at least 1 million.

sovereign government to choose a portfolio of external debt and international reserves. Second, the government is assumed to raise revenue using distortionary taxation. Third, the model includes a sudden stop shock, defined as a complete impediment to borrow. That is, if the economy is hit by this sudden stop shock, the sovereign cannot borrow in the current period and has to repay its debt or choose to default. This feature of the model intends to reinforce the role of financial crises at generating positive demand of reserves for precautionary motives. Fourth, a renegotiation stage is assumed to occur after a sovereign defaults. By allowing for endogenous renegotiation between foreign lenders and the government, the model adequately accounts for realistic debt reductions as observed in the data. Additionally, if international reserves affect the sovereign value of regaining access to international markets, then different levels of reserves will imply different recovery rates for the lenders. This provides another channel influencing demand for international reserves.

The model economy works as follows. A sovereign government from a small open economy chooses distortionary taxes, international reserves, and external debt in order to finance public expenditure and maximize households' welfare. Households supply labor to firms that produce final consumption goods subjected to productivity shocks. However, wages earned by households are taxed by the government. Due to limited commitment in international credit markets, lenders cannot force governments to repay and therefore supply credit with an interest rate spread that reflects an endogenous risk of default. If the government decides to default, domestic productivity suffers a loss and access to credit markets is temporarily barred until debt is restructured. Additionally, if the economy is hit by a sudden stop shock, international lenders loose faith in the government credit for exogenous reasons. When that happens, the government is left with two options: either adjust consumption by repaying the outstanding debt, or default and bear the associated losses.

Given a positive probability of default, the interest rate spread for debt will be positive. Therefore, an impatient government desiring to shift consumption from the future into the present by taking external debt will face a large cost of carrying positive amounts of risk-free interest paying reserves. Reducing simultaneously debt and reserves, implying a constant level of net debt, increases current consumption as the interest rate charged on debt is larger than the one received on reserves. With this cost, it seems unreasonable for a government to hold any debt at all if reserves have no other purpose. However, a choice of no reserves makes the economy more exposed to sudden stops. If the economy is hit by a sudden stop, the government is forced to adjust consumption either by repaying debt or defaulting. In this case, reserves can be used to repay debt, helping the government to prevent a sharp adjustment in consumption and even a default event. The larger the costs associated with the impact of a financial crisis, the more the insurance provided by reserves.[2] On one hand, pro-cyclical taxation, especially recurrent in developing economies (Gavin and Perotti, 1997; or Kaminsky, Reinhart, and Végh, 2005), aggravates such costs as higher taxes distorts output further more. On the other hand, if a government defaults, then current reserves are used during

[2]Recent empirical evidence finds that international reserves reduce the likelihood that a country is hit negatively by global adverse effects, for example, Frankel and Saravelos (2012) or Bussière, Cheng, Chinn, and Lisack (2015).

debt restructuring negotiations thus affecting the recovery rate that lenders face. With reserves providing a limited benefit to a government remaining permanently in autarky, recovery rates will be positively related with the level of reserves. This mechanism provides an additional channel over which a positive level of reserves provides benefits to impatient governments: lenders will convey an expected higher recovery rate into lower interest rate spreads. To summarize, reserves provide two main benefits to the government: an insurance benefit arising from a precautionary motive and a decrease in interest rates from a renegotiation channel.

Using the model outlined above, the goal of this paper is to explain non-trivial levels of reserves and debt to output ratios while verifying other features from the data at the business cycle frequency. The focus on a short-term analysis is related with the fact that international reserves seem to be actively managed by governments during crisis periods. Broner, Didier, Erce, and Schmukler (2013) document dynamics related to debt and reserve accumulation around times of financial crises in emerging market economies, in particular that gross capital inflows and outflows are correlated and both collapse around crises. Additionally, this paper also documents a positive association between international reserves and recovery rates using a dataset compiled by Asonuma and Trebesch (2016) on default episodes and debt restructurings. These two empirical regularities are analyzed by computing a calibrated numerical solution of the model. The resulting simulations are then used to generate moments that are compared against Mexican data. Mexico is chosen as a benchmark to evaluate the model as it is representative of emerging market economies. The dynamics of this country's debt and reserves resemble the one presented in figure 1 and its economy experienced several financial crises in the last forty years.

Link with the literature

This paper is related to the literature of sovereign default of small open economies, with Eaton and Gersovitz (1981) being the classical work. There, the authors present a small open economy dynamic model with non-contingent debt and lack of full commitment that generates equilibrium default. Aguiar and Gopinath (2006) show the quantitative relevance of that model in replicating key business cycle statistics for emerging market economies, in particular, pro-cyclical dynamics of net capital inflows. In a related paper, Arellano (2008) uses a non-linear output loss in the event of a default (increasing in the endowment realization) to generate similar results but with a higher default rate that is in fact closer to what is seen in the data. Using a model similar to Eaton and Gersovitz, Alfaro and Kanczuk (2009) study specifically reserve and debt accumulation by small open economies, concluding that reserves play no role as insurance instruments. Two elements explain their findings. First, a proportional output loss is assumed as the penalty faced by economies that default. As a result, a substantially low discount factor has to be used in order to generate realistic debt to GDP holdings. With such a lower discount factor, savings become almost prohibited in the model. Second, the only risk faced by borrowers in their model is endowment fluctuations. In that case, reserves provide a bad hedge against such risk: market exclusion becomes more bearable when the government holds reserves, thus increasing the spread charged by lenders.

Alternatively, the model presented in this paper uses an output loss similar to Arellano (2008), and adds a sudden stop shock.[3] Jeanne and Ranciere (2011) provides an early model where reserves play a direct role in providing insurance against sudden stops when the country has positive debt holdings. In that paper, a sudden stop is modeled as a persistent event that, on top of the exclusion from markets, entails an output losses on its own. More recently, Bianchi, Hatchondo, and Martinez (2018) use the same idea to generate positive demand for reserves in a model of the Eaton and Gersovitz type when the government debt is long-term. Contrary to those papers, the sudden stop shock version used in this paper does not impact the country in any other way other than a temporary lack of credit access, similar to the sudden stop shock proposed in Roch and Uhlig (2018).

Additionally, this paper includes an endogenous renegotiation channel and distortionary taxation in the model. Yue (2010) studies the interaction between sovereign default and ex post debt renegotiation, concluding that recovery rates are decreasing the the level of debt. Detragiache (1996) and Aizenman and Marion (2004), using simple two-period models, have argued that costly taxation might play an important role in generating demand for reserves when the country faces default risk. Under such circumstances governments would want to prevent not just the direct effect of an output loss generated by a default event, but also the costs of raising revenues when taxation is costly. More recently, Cuadra, Sanchez, and Sapriza (2010) underline that distortionary taxation becomes relevant in an environment of limited risk sharing due to the presence of default. In their model, tax rates increase when output is low, consistent with evidence that developing economies tend to maintain pro-cyclical fiscal policies (Gavin and Perotti, 1997; Kaminsky, Reinhart, and Végh, 2005; Vegh and Vuletin, 2015; and Bauducco and Caprioli, 2014). Tavares (2019) also shows, in the context of a sovereign default model, how pro-cyclical fiscal policies can influence macroeconomic dynamics.

This paper contributes to the literature by providing a framework for studying the dynamics of reserves, debt, and sovereign spreads. Results point to the importance of distortionary taxation and debt restructuring for a realistic quantification of the demand for reserves. To my best knowledge, this paper is the first that can deliver realistic results regarding debt and reserves using a model with one-period debt.[4]

[3] This type of shocks were studied in Calvo (1998).

[4] Samano (2022) presents a sovereign default model with one-period debt that generates realistic levels of external debt and international reserves when a country has an independent central bank that disagrees with the fiscal authority regarding debt accumulation.

Main Results

As a preview of the results presented in later sections, the model used in this paper is able to quantitatively replicate some data moments of the business cycle statistics, such as large debt holdings, default rates, negative co-movement of trade balance and output, and positive correlation between gross capital outflows and inflows. Additionally, strong reserve accumulation is also generated in the simulations. This result stands in contrast with the findings from Alfaro and Kanczuk (2009). Under the proposed model, losses of default are painful enough to generate a large insurance role for reserves. Also of importance is the extension of the standard model to include debt restructuring and costly fiscal collection. Quantitative results from computing and calibrating the model indicate that the baseline model can generate 9% international reserves to GDP as seen in the data for Mexico. If fiscal distortions are shutdown from the model, then only 1% reserves to GDP are sustained in equilibrium, and, if renegotiation is not allowed, less than 0.1% reserves to GDP are generated.

The remaining content of this paper is outlined as follows. Section 2 identifies the main trends in the data and shows empirical relationships between reserves and relevant variables. Section 3 presents the model and defines the equilibrium. Section 4 justifies the functional forms, calibration, and analyzes the simulation results. Section 5 concludes the paper. The numerical procedure and alternative model specifications are contained in appendix A.

2 Empirical Evidence

This paper is mostly concerned with the accumulation of international reserves and external debts. For that purpose, international reserves are defined as external assets held by a country's government or central bank. According to the guidelines of the *International Financial Statistics* (IFS), compiled by the International Monetary Fund (IMF), these assets "comprise holdings of monetary gold, special drawing rights, reserves of IMF members held by the IMF, and holdings of foreign exchange", which include, "foreign banknotes, bank deposits, treasury bills, short- and long-term government securities". As such, most assets considered as reserves are highly liquid and yield interests close to the risk-free rate. It should be noted however that assets managed by sovereign wealth funds, typically yielding higher returns, are not considered international reserves. Different management principles dictate the dynamics of such funds, often characterized by the search of higher returns or different geostrategic values and, for that reason, fall outside the scope of this study.

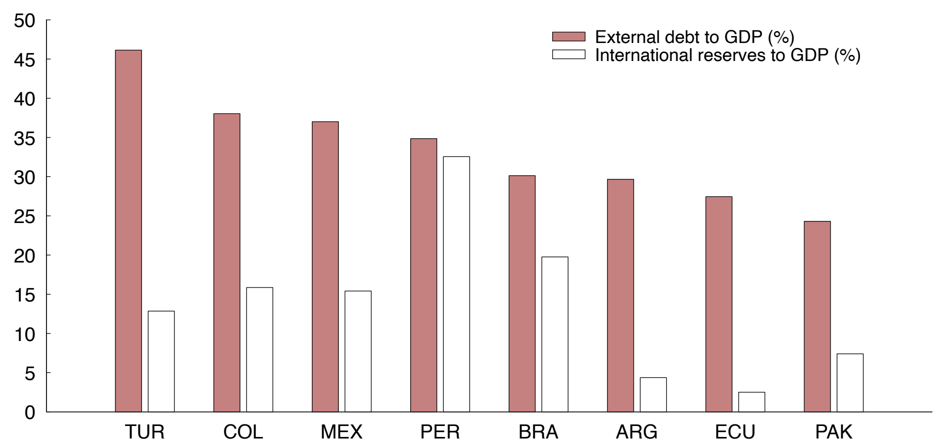
In turn, external debts are defined by the IFS as "external obligations of public debtors, including the national government, political subdivisions (or an agency of either), and autonomous public bodies, and external obligations of private debtors that are guaranteed for repayment by a public entity".[5] Because this paper focuses on the implications of debt default on risk sharing, the debt considered is that owned directly or indirectly by a government, that is by an agent with the option to repudiate it.

Additional details, definitions and sources for all variables used in this paper can be found in section A.1 of the appendix.

What are the costs of holding international reserves?

Part of the reason why a choice of large debt and reserves is intriguing relates to the cost of maintaining such portfolio. As an example, consider a situation where the borrowing interest rate equals i and the savings interest rate equals $i^* > i$. In this case, $i \cdot ND$ is the cost of holding ND units of debt without any reserves. An equal net debt holding can be achieved by borrowing $D = ND + R$ and, at the same time, saving R. The total interest costs of this choice would be $i \cdot (ND + R)$ while $i^* \cdot R$ would be the interest gains. Thus, the cost difference between these two financing options amounts to $(i - i^*) \cdot R$, that is, if the borrowing interest rate is higher than the saving interest rate, holding debt and reserves carries a cost equal to the interest rate spread times the level of reserves.

Figure 2: External debt and international reserves for selected emerging market economies in 2015



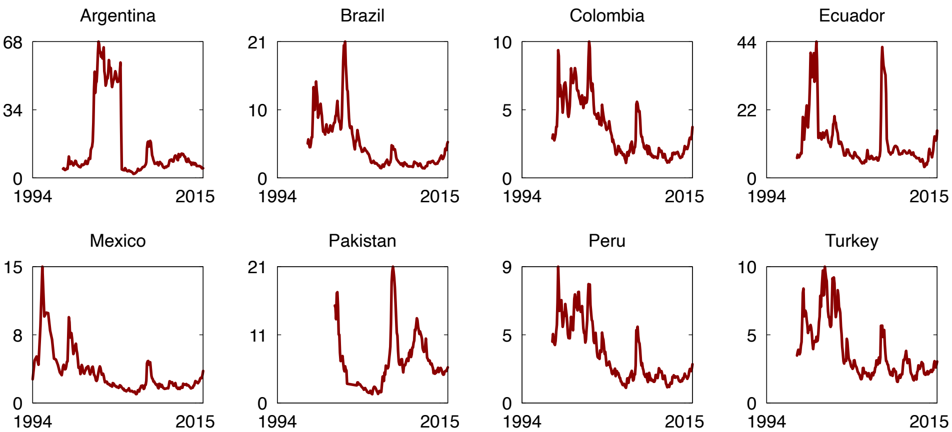
Source: *World Development Indicators*
Notes: All values of external debt and international reserves refer to the year of 2015. The three letters on each pair of columns correspond to the 3 digit country code used in the United Nations.

This example is closely related with the trends observed in emerging market economies. Figure 2 illustrates this fact by showing both levels of external debt and international reserves for a group of selected countries. External debt varies between 25 and 45% of GDP, while international reserves

[5] Due to data limitations from the *World Development Indicators,* public and privately owned external short-term debt are indistinguishable and will be included in the measures of debt used in this paper without distinction.

between 3 and 30%.

Figure 3: Government bond interest rate spreads in selected emerging market economies



Source: *JPMorgan Emerging Market Bond Index* (EMBI)
Notes: The bond spreads presented are defined as a weighted average of yield spreads with respect to US government debt securities for those external debt instruments issued by sovereign entities in emerging market economies that are denominated in US dollars.

With a positive spread between the interest rate on debt relative to reserves, such gap becomes costly. In fact, due to prevalent sovereign debt crises in emerging market countries, spreads have been large as international investors take into account the risk of default when they lend to these countries (see figure 3 for the time series). Spreads[6] are generally quite volatile and high, reaching magnitudes of 20% and larger, even in non-default episode periods.[7] With these facts, a crude estimate on the annual cost of holding reserves can be built as the simple product of the spreads and international reserves. Table 1 shows the calculations: costs can be substantial, oscillating on average between a range of 0.25 to 0.80% annual GDP. Rodrik (2006), using different assumptions, also estimates substantial costs that can be larger than 1% of GDP in some cases.

International reserves impact on interest rate spreads

Given non-negligible costs, one could argue that there must be counterweight benefits to rationalize the observed levels of reserves. This section explores potential benefits of holding international reserves through their influence on interest rate spreads. To evaluate empirically the relationship between spreads and international reserves, this study follows a long literature of regressing spreads

[6]These spreads are given by the secondary market rate, provided by *JPMorgan's Emerging Markets Bond Index* (EMBI), and are measured by an index that includes sovereign and quasi-sovereign (guaranteed by the sovereign) instruments that satisfy certain liquidity criteria in their trading. All spreads are calculated as the premium paid by an emerging market economy over a US government bond with comparable maturity.

[7]The extreme interest rate spread spikes observed in Argentina and Ecuador coincided with default episodes.

Table 1: Crude estimates of costs of holding reserves for selected countries (in % of annual GDP)

Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Argentina	3.65	0.47	0.51	1.10	1.73	0.85	0.60	0.78	0.59	0.47	0.26
Brazil	0.24	0.18	0.23	0.34	0.44	0.26	0.26	0.28	0.30	0.35	0.71
Colombia	0.33	0.18	0.16	0.30	0.35	0.19	0.16	0.15	0.18	0.21	0.40
Ecuador	0.37	0.23	0.47	0.96	1.34	0.35	0.31	0.23	0.29	0.20	0.25
Mexico	0.14	0.11	0.11	0.22	0.34	0.21	0.24	0.26	0.27	0.28	0.39
Pakistan	0.20	0.19	0.29	0.55	0.96	0.61	0.76	0.65	0.24	0.30	0.37
Peru	0.45	0.37	0.38	0.71	0.80	0.52	0.54	0.52	0.52	0.50	0.65
Turkey	0.29	0.26	0.24	0.37	0.43	0.25	0.27	0.38	0.33	0.34	0.34
Mean	0.71	0.25	0.30	0.57	0.80	0.40	0.39	0.41	0.34	0.33	0.42

The entries of this table display a crude calculation of the portfolio cost of maintaining simultaneously external debt and international reserves. This is computed as the product of reserves to GDP with the interest rate spread between debt and reserves.

on covariates[8]. All variables and sources are described in section A.1 of the appendix, and the correlational evidence is based on large panel regressions controlling for country and time effects using annual data. Periods of default and market exclusion are not considered for the analysis. Table 2 reports the results from 3 different commonly used econometric specifications.[9]

All three specifications are consistent at showing that reserves to GDP are negatively associated with spreads while controlling for other variables.[10] As for external debt to GDP, the coefficients across regressions have the opposite sign. These results maintain statistical significance even after controlling for country and time effects. The fixed effects column in table 2 shows that an increase of 10pp of reserves to GDP is associated with an average fall of spreads in the order of 48 basis points or 0.48%. At the same time a 10pp increase of debt to GDP or fall in real GDP of 10% is associated with an increase of spreads of 38 and 67 basis points, respectively. The regression coefficients for the remaining controls have the expected signs, for example, countries under worse budget condition have larger spreads, or economies with better institutions, measured from a rule of law index, tend to be associated with lower spreads. These results update and are consistent with previous empirical studies. For the purpose of this paper, the main message is that international reserves seem to be negatively related with interest rates charged to countries that seek external financing. Such effect can be seen as a potential benefit to hold international reserves.

[8] For example Edwards (1984), Akitoby and Stratmann (2008) or Panizza, Sturzenegger, and Zettelmeyer (2009).

[9] Interest rate spread data from the JPMorgan's EMBI are used for the dependent variable, while controls use data from the *World Development Indicators* (WDI), *World Governance Indicators* (WGI), and the *World Economic Outlook* (WEO). All countries with available observations until 2019 are included, but periods when countries undergo sovereign debt default or restructuring are excluded from the regressions.

[10] To deal with potential sources of contemporaneous endogeneity with spreads, lagged variables were used for: reserves/GDP, debt/GDP, real GDP growth rate, revenues/GDP, expenditures/GDP, and inflation.

Table 2: Effects of international reserves on spreads in cross-country regressions

Dependent variable: Annual mean spread (basis points)	Pooled OLS (I)	Random Effects (II)	Fixed Effects (III)
Main controls:			
Lagged reserves to GDP (%)	-5.258***	-4.819***	-4.846**
	(0.920)	(1.402)	(1.800)
Lagged debt to GDP (%)	4.121***	3.792***	3.849***
	(0.743)	(1.183)	(1.412)
Lagged real GDP growth rate (%)	-2.328	-6.121***	-6.599***
	(3.559)	(2.343)	(2.244)
Other controls:			
Lagged revenues to GDP (%)	-9.996***	-4.367	-1.808
	(3.709)	(5.279)	(7.100)
Lagged expenditures to GDP (%)	13.141***	10.923**	11.786**
	(3.850)	(4.153)	(5.520)
Lagged inflation rate (%)	6.091***	5.925***	5.985***
	(2.136)	(1.474)	(1.727)
Trade openness†	0.281	1.143**	2.075*
	(0.351)	(0.528)	(1.041)
Contagion†	0.103	0.220**	0.270***
	(0.071)	(0.082)	(0.083)
Rule of law†	-301.352***	-340.819***	-380.563***
	(40.414)	(57.047)	(76.729)
Urban population†	-0.514	-2.211	0.494
	(0.937)	(2.041)	(8.779)
Time effects	Yes	Yes	Yes
Number of countries	37	37	37
Observations	359	359	359
R^2	0.54	0.52	0.48

Notes: The table presents regression coefficients of the annual interest rate spread against different controls. The use of lagged variables refers to the observation on the same country in the previous year. In column (I) the statistical model is estimated using simple pooled OLS, in column (II) using a standard random effects model; and in column (III) a fixed effects model. All regressions include common time fixed effects. The number in parenthesis correspond to robust standard errors.

† Section A.1 of the appendix provides detailed information for of each of these variables.

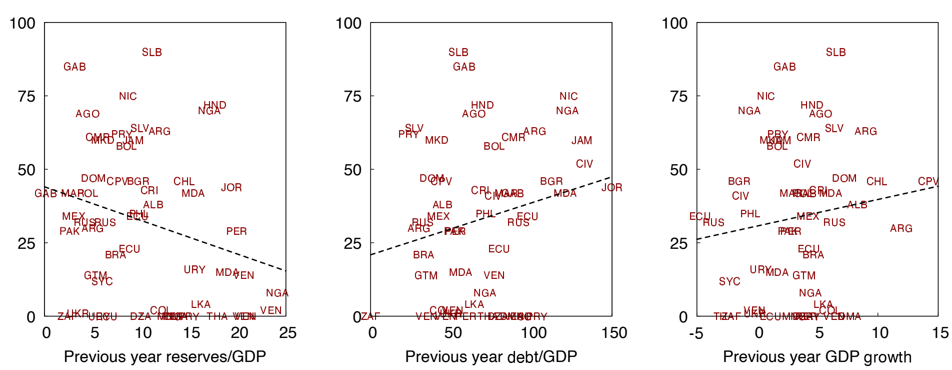
* significance at 10%; ** significance at 5%; *** significance at 1%

International reserves and haircuts

Rational investors lending to a country should price the interest rate taking into account the probability of repayment and, in case of a default, the expected recovery rate of the overdue debt. Previous empirical studies on the determinants of interest rate spreads, for example Edwards (1984) or Akitoby and Stratmann (2008), assume that such expected recovery rate is zero, implying that regression coefficients similar to the ones presented in table 2 could be directly mapped into default probabilities. However, with non-zero expected recovery rates, these coefficients may confound two different effects of the related variable: the probability of default and the expected recovery rate. This implies that a more complete analysis of the relationship of reserves on spreads should also take into account how recovery rates relate with reserves.

To study how recovery rates or haircuts (an equivalent term defined as the complement of the recovery rate) are related with external debt and international reserves, a dataset on historical haircuts associated to sovereign default episodes compiled by Benjamin and Wright (2009) is used. These estimates have become commonly used in the literature studying restructuring of sovereign defaulted debt.[11] Figure 4 shows how haircuts relate to lagged reserves, debt and the GDP growth rate. The scatter plots suggest a negative relationship between haircuts and reserves, that is, the higher the level the reserves, the larger is the recovery rate on the defaulted debt. At the same time, countries with larger debt to GDP tend to have large haircuts and the GDP growth rate has a positive but low correlation with haircuts.

Figure 4: Haircuts on default episodes for middle income countries



Source: *World Development Indicators* and Benjamin and Wright (2009).
Notes: The dashed line in the figure is a linear fit between debt haircuts and the variable of interest.

This observation is reinforced when one uses simple multivariate regression techniques.[12] Table 3 presents results of cross-country regressions of haircut rates at the time of sovereign debt restructuring against lagged values for reserves, debt, and the GDP growth rate. Data for the haircut rates are taken from Benjamin and Wright (2009) and Cruces and Trebesch (2013), two datasets commonly used in this literature. Additional controls include the time that spans between the period when a government defaults and when it reaches a restructuring agreement with the lenders. The estimated regression coefficients show a negative and statistically significant association of lagged

[11]Examples include Yue (2010) and Erasmo (2008). The first studies how haircuts change with the level of debt and the second studies how haircuts are related with delays in restructuring.

[12]More elaborate regressions that take advantage of the panel structure of the data were not used due to small number of default observations per country.

reserves even after controlling for lagged debt or output growth. In particular, a 10pp increase in lagged reserves to GDP is associated with a fall in external debt haircuts of 15 to 12pp upon a default event. Such results suggest that the accumulation of reserves can improve the lenders' outcome if a default is eventually triggered by a government.

Table 3: Effects of international reserves on haircuts rates (% of overdue debt)

Dependent variable: Haircut rate at restructuring (%)	Benjamin and Wright (2009) (I)	Cruces and Trebesch (2013) (II)
Main controls:		
Lagged reserves to GDP (%)	-1.502***	-1.214***
	(0.364)	(0.447)
Lagged debt to GDP (%)	0.584***	0.419***
	(0.108)	(0.131)
Lagged real GDP growth rate (%)	-0.302	0.208
	(0.231)	(0.437)
Other controls††:		
Lagged inflation rate (%)	-0.235***	-0.003
	(0.0555)	(0.004)
Trade openness†	-0.252***	0.357**
	(.0829)	(0.141)
Urban population†	0.465***	0.493***
	(0.135)	(0.137)
Time delay to restructuring	-1.965	8.156***
	(2.868)	(2.670)
Observations	31	79
R^2	0.85	0.74

Notes: The table presents simple OLS regression coefficients of haircut rate spread against different controls. The haircut rate refers to the loss that lenders agree to accept in debt restructuring negotiations. The use of lagged variables refers to the observation on the same country in the previous year. Column (I) uses the historical data on sovereign debt restructuring compiled by Benjamin and Wright (2009) while column (II) uses data compiled by Cruces and Trebesch (2013). Large periods that span between default and restructuring are restricted to at most 6 years, but the results are robust to increase of decrease this threshold. The numbers in parenthesis correspond to robust standard errors.

† Section A.1 of the appendix provides detailed information for each of these variables.

†† The variables on government revenues, expenditures, contagion, and rule of law presented in table 2 are excluded from these regressions as they would reduce even further the number of observations.

* significance at 10%; ** significance at 5%; *** significance at 1%

3 Model

In light of the evidence presented, a model economy is introduced in this section where the optimal choice of reserves depends on the trade-off between the cost of holding reserves, closely related with interest rate spread, and the benefits of holding reserves, linked to three different motives: smoothing the impact of a crisis on consumption, minimizing fiscal distortions on production, and changing the negotiation position if a default episode occurs. These last two benefits represent the main innovation that is introduced here relative to previous models from the literature. They provide a better accounting of the observed dynamics regarding external debt and international reserves.

The model builds on classical work from Eaton and Gersovitz (1981), and recent applications in Aguiar and Gopinath (2006) and Arellano (2008).[13] A small open economy is populated with a representative household, a firm, and a government. The household is 'hand-to-mouth', simply consuming any income net of taxes earned in each period. This assumption mutes domestic credit markets to highlight the role of external debt and international reserves markets. The firm buys labor to produce final goods with a production function that is subjected to diminishing returns and to a multiplicative technology shock. The government acts on behalf of the household by making decisions about the amount of debt and reserves to hold, accessible in international markets. It also taxes consumers in order to finance public expenditures which, for the sake of simplicity, are exogenous. Due to limited commitment in the enforcement of debt contracts, the government can default on its own debt. Under such scenario, access to international markets is temporarily shut down and a renegotiation process follows. While excluded, firms suffer a loss in productivity and lenders recover part of the repudiated debt by allowing the government to regain access to external credit markets. Additionally, in any time period, lenders can loose confidence in debtors for exogenous reasons. A sudden stop then echoes as an impediment to the renewal of the government's loans.

3.1 Household

An infinite lived representative household values lifetime consumption and labor according to:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_t, h_t) \tag{1}$$

where $\mathbb{E}$ is the expectation operator, β denotes the discount factor, and the period utility $u : \mathbb{R}_+ \times [0,1] \to \mathbb{R}$ is: continuous, differentiable and concave in both arguments, increasing in consumption c_t, and decreasing in hours h_t.

The household supplies labor to the firm at a wage rate w_t, but this income is taxed at a rate τ_t. No savings are allowed and profits π_t are transferred as lump sum. Income earned is used for consumption, yielding the following time t budget constrain:

$$c_t = (1-\tau_t)w_t h_t + \pi_t \tag{2}$$

Optimal household behavior regarding consumption and supply of hours can therefore be char-

[13] A simple two-period version of this model with some implications is introduced in section A.2 of the appendix.

acterized by equation (2) and the following first order condition:

$$-\frac{u_h(c_t, h_t)}{u_c(c_t, h_t)} = (1 - \tau_t)w_t \tag{3}$$

3.2 Firm

The firm in the economy maximizes profits by using hours h_t in a production technology subjected to diminishing returns $f(h_t)$ and random productivity shocks z_t. The function $f : [0, 1] \to \mathbb{R}$ is continuous, differentiable, concave and satisfies the Inada conditions. The productivity shock z_t evolves accordingly to a Markov process. Profits at time t are given by:

$$\pi_t = z_t f(h_t) - w_t h_t \tag{4}$$

From maximization, these imply the following labor demand condition:

$$w_t = z_t f_h(h_t) \tag{5}$$

3.3 Government

Acting on the household's behalf, the government with a good credit history has the ability to borrow or save in international markets. Due to incompleteness of financial markets, the government can only borrow by selling non-contingent one-period bonds D_t. At the same time, it can also choose to save by buying one-period international reserves R_t. To finance public expenditures g_t, it can also tax wages. If the government decides to repudiate its own debt, credit history becomes bad with further exclusion from international credit markets. In this situation, firms productivity becomes $\tilde{z}_t = z_t - l(z_t)$, where $l(z_t)$ is a continuous loss function such that $0 \leq l(z_t) \leq z_t$. Putting all these elements together, the government budget constraint in period t is given by:

$$\tau_t w_t h_t = g_t + D_t - q_t D_{t+1} - R_t + \bar{q} R_{t+1}, \quad \textit{with good credit history} \tag{6}$$

$$\tau_t w_t h_t = g_t - R_t + \bar{q} R_{t+1}, \qquad \textit{otherwise} \tag{7}$$

where q_t is the endogenous price of new debt and $\bar{q}$ the risk-free price of new international reserves.

3.4 International investors

International investors provide debt and reserve assets to the government. However, the economy can suffer a random sudden stop shock denoted by s_t. Similarly to Roch and Uhlig (2018), s_t is interpreted as a "crisis" sunspot where, for extraneous reasons, the government looses access to international markets.[14] In this model the sudden shock realization is independent from all other

[14]For a model where sunspots can generate large shifts in the borrowing conditions see Cole and Kehoe (2000).

variables, taking the value $s_t = 1$ if the economy is hit by an external loss of confidence, or $s_t = 0$ otherwise. If the economy is in a sudden stop, no international investor provides credit to the economy. This is equivalent as stating that the price of new debt equals zero. If instead the economy is not in a sudden stop, international investors price debt so that, in expectation, their profits are zero. In the event of a default, it is assumed that they recover an amount $\tilde{\varphi}(D, R, z) \in [0, D]$ of the arrears debt in the period following renegotiations. Note that under this case, the recovery rate schedule is given by $\tilde{\varphi}(D_t, R_t, z_t)/D_t$. Letting $\tilde{Z}(D_t, R_t, z_t, s_t)$ be an indicator function taking 1 if the government defaults and 0 otherwise, new debt becomes priced as:

$$q(D_{t+1}, R_{t+1}, z_t, s_t) = \begin{cases} 0, & if\ s_t = 1 \\ \tilde{q}(D_{t+1}, R_{t+1}, z_t), & otherwise \end{cases} \tag{8}$$

where

$$\begin{aligned} \tilde{q}(D_{t+1}, R_{t+1}, z_t) =& \bar{q} \cdot \left\{ \int \left[1 - \tilde{Z}(D_{t+1}, R_{t+1}, z_{t+1}, s_{t+1}) \right] dF(z_{t+1}, s_{t+1} | z_t) \right. \\ & \left. + \int \tilde{Z}(D_{t+1}, R_{t+1}, z_{t+1}, s_{t+1}) \cdot \bar{q} \frac{\tilde{\varphi}(D_{t+1}, R_{t+1}, z_{t+1})}{D_{t+1}} dF(z_{t+1}, s_{t+1} | z_t) \right\} \end{aligned} \tag{9}$$

and it is assumed that (z_t, s_t) evolve according to the transition probability given by $F(z_{t+1}, s_{t+1} | z_t,)$. From the definition of (8), the price schedule is bounded by $q_t \in [0, \bar{q}]$, implying that interest rates on borrowing $(1/q_t - 1)$ are always equal or larger than the risk-free rate $(1/\bar{q} - 1)$.

3.5 Timing

The events characterizing this model can be structured with the following order. At the beginning of period t a government with good credit history:

1. Starts period t with debt and reserves levels of D_t and R_t respectively.
2. Sudden stop and productivity shocks are realized: s_t and z_t.
3. The government decides whether or not to default.
   (a) If the government decides not to default:
      i. Choses D_{t+1}, and R_{t+1} at prices $q(D_{t+1}, R_{t+1}, z_t, s_t)$. The remaining variables $(c_t, h_t, \tau_t, w_t, \pi_t)$ are determined by the model's agents.[15]
      ii. Advances to period $t + 1$ with a good credit history carrying debt and reserves levels of D_{t+1} and R_{t+1}.

[15] Note that when D_{t+1} and R_{t+1} are chosen, τ_t is determined from equations (3), (5), (6) and (7).

(b) If the government decides to default:

i. The country enters in financial autarky. The government still chooses R_{t+1} and the remaining variables $(c_t, h_t, \tau_t, w_t, \pi_t)$ are determined by the model's agents. At the same time, the government negotiates how to restructure its debts with lenders, agreeing to pay $\tilde{\varphi}(D_t, R_t, z_t)$ in the next period.

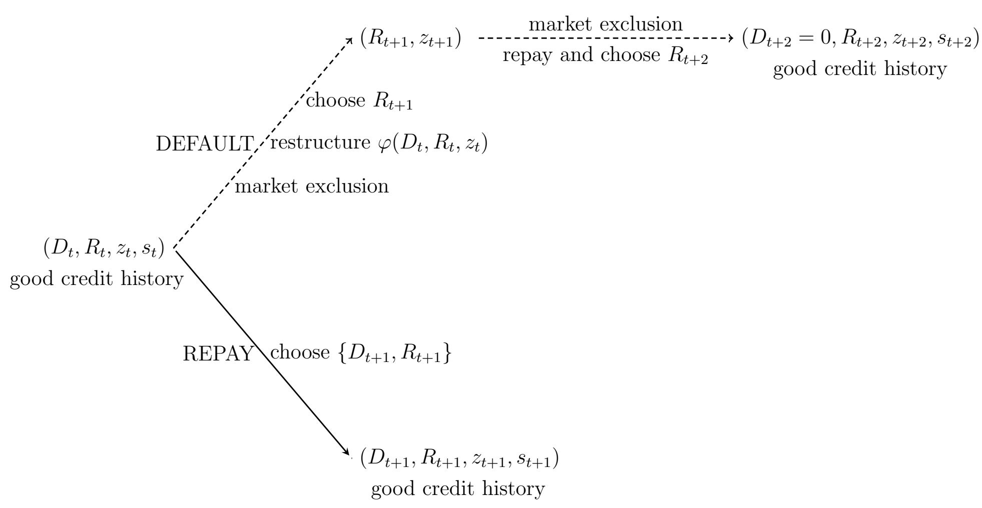
ii. Advances to period $t+1$ still with bad credit history. The government pays $\varphi(D_t, R_t, z_t)$ and decides on R_{t+2}. The remaining variables are determined $(c_t, h_t, \tau_t, w_t, \pi_t)$ by the model's agents.

iii. Advances to period $t+2$ with a good credit history, no debt and a reserves level of R_{t+2}.

Given that the focus of this paper is not in the study of delays in debt restructuring, it is assumed in the model that restructuring is exogenously resolved in the period that follows a default. Given this framework, the government must negotiate a debt restructuring with lenders when it defaults and, in the next period, transfer the agreed amount. Despite being restrictive, this environment generates results that are similar to the ones obtained in models of renegotiation such as Yue (2010) or Erasmo (2008) where agents choose to renegotiate very quickly, even though they are specifically allowed to delay repayments or renegotiations.

For convenience, the timing of the model is also schematized in figure 5.

Figure 5: Sequence of events implied in the model



3.6 Recursive formulation of the problem

The government's problem consists on maximizing consumers utility given by (1) subject to all the constraints summarized in equations (2) to (8). Let v^{rep} be the value for a government who repays its debt, and v^{def} the value for a government that defaults. The state variables at the beginning of the periods are (D, R, z, s). Then, the previous problem can be represented recursively as:

$$v^{rep}(D,R,z,s) = \max_{D',R'} \left\{ u(c,h) + \beta \mathbb{E}_{z,s} \left[\max \left\{ v^{rep}(D',R',z',s'), v^{def}(D',R',z') \right\} \right] \right\} \tag{10}$$

$$st$$

$$c = zf(h) - g - D + qD' + R - \bar{q}R' \tag{11}$$

$$-\frac{u_h(c,h)}{u_c(c,h)} = zf_h(h) - \frac{g + D - qD' - R + \bar{q}R'}{h} \tag{12}$$

The first constraint (11), resulting from combining equations (2), and (4)-(6), is a resources constraint: private and public consumption equate to the sum of domestic production plus net external inflows, independently of these flows being positive or negative. The second constraint (12), which combines equations (2)-(6), is a labor market equilibrium condition. That is, it represents the set of competitive allocations (c, h) such that both consumers and firms are optimizing given prices and taxes. Note also that, under this representation, tax rate is no longer explicitly present in the equations. This is because they are determined by the next period choice of debt and reserves. As such, the government's problem collapses into choosing the level of reserves and debt for the next period subject to both resources and labor market constraints. Given an allocation (c^*, h^*, D'^*, R'^*) that solves (10), tax rates can be recovered from equations (3) and (5) as:

$$(1-\tau) = -\frac{u_h(c^*,h^*)}{u_c(c^*,h^*)} \cdot \frac{1}{zf_h(h^*)}$$

If, instead, the government decides not to repay, it spends one additional period in financial autarky that will be used to reach an agreement with its creditors. Given these elements, the government's value of default can be defined as:

$$v^{def}(D,R,z) = \max_{R'} \left\{ u(c,h) + \mathbb{E}_z \left[\max_{R''} \left\{ \beta u(c') + \beta E_{z'} v^{rep}(0, R'', z'', 0) \right\} \right] \right\} \tag{13}$$

$$st$$

$$c = \tilde{z}(z)f(h) - g + R - \bar{q}R'$$

$$\frac{u_h(c,h)}{u_c(c,h)} = \tilde{z}(z)f_h(h) - \frac{g - R + \bar{q}R'}{h}$$

$$c' = \tilde{z}(z')f(h') - g + R' - \bar{q}R'' - \tilde{\varphi}(D,R,z)$$

$$\frac{u_h(c',h')}{u_c(c',h')} = \tilde{z}(z')f'_h(c',h') - \frac{g - R' + \bar{q}R''}{h'}$$

$$R' \leq R$$

The first and second pair of equations represent the resource constraints in the first and in the second period, before the government is readmitted into international credit markets. In the first period of default, the government suffers a productivity loss $\tilde{z}(z)$ and agrees to repay $\tilde{\varphi}(D, R, z)$ to his creditors in the next period. In order to regain access to international markets the government has to repay his agreed debt. Note that the last constraint entails that the government cannot increase reserves while in autarky. This assumption prevents the government from accumulating too much reserves in the period that immediately precedes restructuring. A more complete model with a longer and uncertain period of market exclusion would not require such constraint.

3.7 Renegotiation

As in Yue (2010), if the government defaults and becomes excluded from international markets, then renegotiation follows immediately. Borrowers and lenders bargain over a recovery amount in exchange for access to international credit markets. If negotiations fail, the government becomes forever excluded from international credit markets. Assuming that all bargaining power is at the side of the lenders, the problem simplifies to a one-shot offer that the borrower can either accept or reject. This negotiation structure delivers similar qualitative and quantitative results as one where the bargaining power is split between lenders and the borrower given an appropriate calibration.

Let $v^{aut}(R, z)$ be the value of permanent autarky, and $v^{rep}(0, R, z, s)$ the value of being in the market with zero debt. For a government in default, holding R, and with productivity z, reaching an agreement for a recovery amount φ has a value given by:

$$\Lambda^G(\varphi; R, z) = \max_{R'<R} \left\{ u(c, h) + \mathbb{E}_{z,s} \left[\max_{R''<R'} \left\{ u(c', h') + \beta \mathbb{E}_{z',s'} \left[v^{rep}(0, R'', z'', s'') \right] \right\} \right] - v^{aut}(R, z) \right\}$$

$$\begin{aligned}
&st \\
&c = \tilde{z}(z) f(h) - g + R - \bar{q} R' \\
&\frac{u_h(c, h)}{u_c(c, h)} = \tilde{z}(z) f_h(h) - \frac{g - R + \bar{q} R'}{h} \\
&c' = \tilde{z}(z') f(h') - g + R' - \bar{q} R'' - \varphi \\
&\frac{u_h(c', h')}{u_c(c', h')} = \tilde{z}(z') f'_h(h') - \frac{g - R' + \bar{q} R''}{h'} \\
&R^{'} \leq R
\end{aligned}$$

where $z - \tilde{z} = l(z) \geq 0$ represents the output loss of exclusion. The outside option for the government under permanent autarky has a value of:

$$v^{aut}(R,z) = \max_{R'} \left\{ u(c,h) + \beta \mathbb{E}_z \left[v^{aut}(R',z') \right] \right\} \tag{14}$$

$$st$$

$$c = a\tilde{z}(z)f(h) - g + R - \bar{q}R' \tag{15}$$

$$\frac{u_h(c,h)}{u_c(c,h)} = \tilde{z}(z)f_h(h) - \frac{g - R + \bar{q}R'}{h} \tag{16}$$

where $z - \tilde{z} = l(z) \geq 0$ represents the output loss of exclusion. The outside option for the government under permanent autarky has a value of:

$$v^{aut}(R,z) = \max_{R'} \left\{ u(c,h) + \beta \mathbb{E}_z \left[v^{aut}(R',z') \right] \right\} \tag{17}$$

$$st$$

$$c = a\tilde{z}(z)f(h) - g + R - \bar{q}R' \tag{18}$$

$$\frac{u_h(c,h)}{u_c(c,h)} = \tilde{z}(z)f_h(h) - \frac{g - R + \bar{q}R'}{h} \tag{19}$$

where $a > 0$ is a parameter that further regulates the productivity level of a country that enters permanent autarky. This parameter is used later to calibrate the recovery rate from the data.

It follows that the problem for the lenders consists in offering a recovery amount proposal φ to the borrower that satisfies its participation constraint given by $\Lambda^G(\varphi; R, z) \geq 0$. That optimal offer $\tilde{\varphi}$ can be summarized as the solution of the following problem:

$$\tilde{\varphi}(D,R,z) = \arg \max_{0 \leq \varphi \leq D} \{\bar{q}\varphi\} \tag{20}$$

$$st$$

$$\Lambda^G(\varphi; R, z) \geq 0$$

3.8 Recursive Equilibrium

All elements are now available to define a stationary recursive equilibrium in this model economy. The equilibrium notion is of a Markov Perfect Equilibrium, that is, policy actions about debt, reserves, default, and negotiation depend only on pre-determined relevant state variables.

Definition 1. A recursive equilibrium is a set of:

i) Value functions: $v^{def}(D,R,z,s)$ and $v^{aut}(R,z)$

ii) Debt price function: $q(D',R',z,s)$

iii) Debt recovery function: $\tilde{\varphi}(D,R,z)$

such that:

a) Given the debt price function $q(D', R', z, s)$ and the debt recovery function $\tilde{\varphi}(D, R, z)$, the value function $v^{rep}(D, R, z, s)$ solves the government problem (10)

b) Given the value function $v^{rep}(D, R, z, s)$ and the debt recovery function $\tilde{\varphi}(D, R, z)$, the debt price function $q(D', R', z, s)$ is consistent with the lenders zero profit condition in (8)

c) Given the value functions of repayment $v^{rep}(D, R, z, s)$, autarky $v^{aut}(R, z)$ and the debt price function $q(D', R', z, s)$, the debt recovery function $\tilde{\varphi}(D, R, z)$ solves the debt renegotiation problem (20)

4 Calibration and quantitative analysis

To analyze the quantitative properties of the model introduced in the preceding section, functional forms are chosen and a calibration is proposed for the numerical computation. The model is used to evaluate the role of international reserves when the debt choice is endogenous and willingness-to-pay incentives becomes a function of the sovereign's choices.

4.1 Functional forms

The numerical implementation of the model uses a utility function of the form proposed by Greenwood, Hercowitz, and Huffman (1988):

$$u(c, h) = \frac{1}{1-\sigma} \cdot \left(c - \Gamma \frac{h^{1+\gamma}}{1+\gamma}\right)^{1-\sigma} \tag{21}$$

This utility function has the advantage of shutting down the wealth effect on labor supply, therefore shocks in the productivity process have an output response of the same signal.[16]

The production technology is just given by a Cobb-Douglas function:

$$f(h) = h^{\alpha}$$

where α is the elasticity of output with respect to labor.

Regarding the output level in the event of a default, Arellano (2008) showed that a non-linear function that induces a disproportionally larger loss if the country defaults in an expansion is important to allow for realistic default probabilities. Also, large output contractions at defaults followed by recoveries (coinciding with credit market re-access) have been documented, for example, in Yeyati and Panizza (2011). Protracted losses in output are explained with disruptions of credit

[16]This utility function has a long tradition in literature studying business cycles in small open economies, for instance, Mendoza (1991), Neumeyer and Perri (2005) or Aguiar and Gopinath (2007).

flows to the private sector that prevents normal production (Mendoza and Yue, 2012). As such, a similar functional form is assumed as appropriate for the productivity under default in this model economy:

$$\tilde{z}(z) = \begin{cases} z & if\ z \geq \hat{z} \\ \hat{z} & otherwise \end{cases} \tag{22}$$

The sudden stop shock is added in order to induce the government to hold reserves as insurance against exogenous shut downs of credit markets. This idea was first introduced in Jeanne and Ranciere (2011) and applied more recently in Bianchi, Hatchondo, and Martinez (2018). In those models, governments hold reserves as a buffer not just against rollover risks, but also against direct output costs that come along with a sudden stop. This paper assumes a milder version of a sudden stop shock that does not impact the economy other than in the momentarily exclusion from credit markets. Additionally, the sudden stop shock is modeled as being independent and identically distributed in every period. The corresponding probability associated with a sudden-stop shock is then assumed to be $Prob\,(s_t = 1) = \omega$.

Finally, as commonly used in the literature, the productivity process is modeled as a log-normal $AR(1)$, with

$$\log z' = \rho_z \log z + \epsilon', \quad \epsilon' \sim N(0, \sigma_z)$$

This process is discretized into a 35 state Markov chain using the Tauchen (1986) method with bounds given by $\log z \in \left[-5.5 \cdot \sigma_z/\sqrt{1-\rho_z^2};\ 5.5 \cdot \sigma_z/\sqrt{1-\rho_z^2}\right]$.

The model is numerically solved using value function iteration where relevant functions are interpolated to allow for choices outside of the chosen grid.[17] A detailed explanation of the algorithm and numerical methods used can be found in section A.3 of the appendix.

4.2 Parameters and calibration

The model is computed at an annual frequency. Then, the solution is used to evaluate the model's ability to generate large government choices of debt and reserves while allowing the economy to match other features of the data such as default rates and cyclical properties of consumption, trade balance, or interest rates. For this exercise, Mexico is used as a reference for the parameter's choice. As a representative country from the set of emerging economies Mexico has an additional advantage of having available data at an annual frequency for a long period ranging from 1960 to 2019.[18] Moreover, the Mexican economy experienced a sovereign default episode in 1983 (after a

[17] This is due to the recommendation of Hatchondo et al. (2010) about solving numerically sovereign default models.

[18] Data for the Mexican aggregate macroeconomic variables use the national accounts dataset provided by the OECD.

collapse of world commodity prices) and a near default in 1994 (when the country was rescued by IMF and the US Treasury). Mexico has also been displaying strong dynamics in the accumulation of both debt and reserves: in the last two decades the government more than doubled its holdings in international reserves to about 10% of GDP while keeping an external debt level to GDP ratio close to 30%.

All the data referring to Mexico are annual real series obtained from OECD, except for external debt and international reserves that are taken from the World Development Indicators at an annual frequency. Output and private consumption are in logs and the trade balance is presented as a percentage of GDP. Following the methodology proposed by Mendoza, Razin, and Tesar (1994), an effective tax rate is computed for the Mexican economy.[19] The interest rate spreads corresponds to the EMBI for 1994–2007 and all other series are from 1960 to 2019. All series are filtered with a Hodrick–Prescott filter to remove the time-trend component.

Table 4 lists the parameters used in the baseline solution of the model. The table is divided in a first set of parameters taken directly from the data or the literature, and a second set that uses the model simulated moments to infer the parameter values. The two first parameters refer to the utility function of the household. The parameter σ, the risk aversion coefficient, is set to 2 which is a standard value used in the quantitative macroeconomics literature. Given the range of estimates available in the literature (for example in Greenwood, Hercowitz, and Huffman, 1988), a value of 0.5 is picked for the Frish elasticity, that is, $\gamma = 2$.

In Jeanne and Ranciere (2011), a sudden stop corresponds to episodes when capital inflows to GDP, measured by the current account, falls by more than 5 percent of GDP relative to the previous year. They then verify that two sudden stops occurred for the Mexican economy in a period spanning 60 years. This estimate gives a 3.5% probability of being hit by a sudden stop, a value that is also used to set ω.

As common in the literature of sovereign default studies, the risk-free bond price $\bar{q}$ is set to 1/1.02, intended to capture the historical average of 2% real interest rate of a five-year US treasury bond. As for the parameter α, the output elasticity with respect to output, its value is taken directly from OECD estimates on the labor income share which averages 0.5 for the Mexican economy.

The remaining parameters are jointly calibrated to match certain moments of the data. An observed average of 15% public consumption to GDP is targeted with the aid of setting $g = 0.09$. Due to lack of enough data on haircuts for Mexico's default episodes, the recovery rate average of 55% presented in the Benjamin and Wright (2009) dataset is targeted. The calibration procedure generates a parameter value of $a = 0.999$. Both parameters governing the productivity shock are calibrated simultaneously as output dynamics are not directly inherited from the productivity z due to endogenous labor supply. The targeted data moments are the standard deviation and autocorrelation of GDP given by $stdev(GDP) = 0.032$ and $corr(GDP_t, GDP_{t-1}) = 0.63$ with

[19] Section A.1 of the appendix shows how these estimates are computed and specifies data sources.

Table 4: Calibrated and set parameter values used in the model's simulations

Parameters		Value	Target
Externally set:			
Risk aversion	σ	2	Standard in the literature
Inverse Frish elasticity	γ	2	Standard in the literature
Risk-free debt price	$\bar{q}$	1/1.02	US government real interest rate
Probability of sudden stop	ω	0.035	Jeanne and Ranciere (2011)
Output elasticity of labor	α	0.5	Labour income share in GDP (Mexico)
Internally calibrated:			
Productivity shock persistency	ρ_z	0.8500	GDP volatility and autocorrelation (Mexico)
Productivity shock volatility	σ_z	0.0210	GDP volatility and autocorrelation (Mexico)
Discount factor†	β	0.7967	Average interest rate spread (Mexico)
Productivity cost	$\hat{z}$	0.9233	Debt/GDP and volatility of trade balance (Mexico)
Productivity under autarky	a	0.9994	Recovery rate in Benjamin and Wright (2009)
Government spending	g	0.0900	Average government spending to GDP (Mexico)
Disutility of labor	Γ	5.3642	Average hours of 1/3

Notes: The first five parameters are set exogenously using values that are commonly used in the literature or set directly to capture features of the Mexican economy. The remaining 7 parameters are set through a calibration procedure that target Mexico's business cycle statistics and statistics related to recovery rates observed in historical sovereign debt defaults. These are jointly estimated to target simultaneously all moments associated with the last column. For that reason, each line association between the moment and the data target is merely suggestive.

† The calibrated discount rate of $\beta = 0.797$ corresponds to a quarterly discount rate of 0.945 a value that is not uncommon to observe in the sovereign default literature (for example, Arellano, 2008 uses a discount rate of 0.95 in a model of quarterly frequency).

correspondent parameter values of $\sigma_z = 0.021$ and $\rho_z = 0.85$. The remaining two parameters $\{\beta, \hat{z}\}$ are also simultaneously calibrated to target the following additional data moments: mean debt to GDP, the standard deviation of the trade balance, and the average interest rate spread. Debt to GDP is targeted to be 37% in the simulations, the standard deviation of the trade balance to 1.9%, and the interest rate spread to an average of 2.5%. The corresponding calibrated values for the parameters amount to $\beta = 0.797$ and $\hat{z} = 0.9233$.

4.3 Simulation results

This section compares the quantitative predictions of the model against observed data. To that end, an economy using table 4 parameters is computed and simulated by averaging the moments of interest for a sample running 10000 periods, where the first 150 are discarded to reduce the influence of initial conditions. These moments are computed for all periods while the government is not in default with the four initial years after a market exclusion being also excluded. Table 5 reports the results of the exercise. The standard deviations referred in the table are expressed in percentage points; y and c refer to the log of output and total consumption of de-trended series; $D^{recover}/D$ is the recovery rate faced by lenders at a debt restructuring; the default rate is an annual rate; trade balance TB/Y is defined as the difference between output and total consumption relative to output; D/Y and R/Y are, respectively, the external debt and international reserve level expressed as a percentage of output; G/Y refers to government spending to GDP; the annual interest spread is given by $i^{spread} = \bar{q}/q - 1$.

Table 5: Business cycle statistics for the benchmark model and data

		Data	Model
Targeted moments:			
Mean debt to GDP	$\mathbb{E}(D/Y)$	37%	38%
Mean interest rate spread	$\mathbb{E}\left(i^{spread}\right)$	2.5%	2.5%
S.D. of trade balance to GDP	$\mathbb{SD}(TB/Y)$	1.9%	2.4%
S.D. of GDP	$\mathbb{SD}(y)$	3.2%	3.9%
Autocorrelation of GDP	$\mathbb{AC}(TB/Y)$	0.63	0.67
Mean debt recovery rate	$\mathbb{E}(D^{recov}/D)$	55%	52%
Mean government consumption to GDP	$\mathbb{E}(G/Y)$	15%	16%
Non-targeted moments:			
Mean reserves to GDP	$\mathbb{E}(R/Y)$	8%	9%
S.D. of the interest rate spread	$\mathbb{SD}\left(i^{spread}\right)$	0.9%	0.7%
Relative volatility of consumption with respect to GDP	$\mathbb{SD}(c)/\mathbb{SD}(y)$	1.4	1.5
Correlation of consumption with GDP	$\mathbb{C}(c, y)$	0.93	0.96
Correlation of the interest rate spread with GDP	$\mathbb{C}\left(i^{spread}, y\right)$	-0.39	-0.46
Correlation of the trade balance with GDP	$\mathbb{C}(TB/Y, y)$	-0.60	-0.55
Correlation of the tax rate with GDP	$\mathbb{C}(\tau, y)$	-0.49	-0.74
Default rate	$\mathbb{E}(\tilde{Z})$	2-5%	4.6%

Notes: The business cycle statistics described in the last column are generated by simulating the economy for 10000 periods, where the first 150 are discarded to reduce the influence of initial conditions. In the second column, the functions $\mathbb{E}$, $\mathbb{SD}$, $\mathbb{AC}$, and $\mathbb{C}$ correspond to the mean, standard deviation, autocorrelation, and correlation, respectively. The data default rate statistic $\mathbb{E}(\tilde{Z})$ captures a range of historical sovereign debt default episodes as documented in Reinhart and Rogoff (2009).

A first impression from the results is that the simulated economy can replicate key features of the data. The model delivers a mean debt to output ratio of 38% together with 9% for reserves to output. Large values of debt to output are sustained in equilibrium due to intense losses in a default event. Default losses are associated with exogenous losses of productivity given by the function (22) and an endogenous fiscal adjustment that distorts production by reducing labor supply. This is in contrast with Arellano (2008) that, to target the default rate observed in Argentina, uses a milder and completely exogenous output loss, ending up with a model that generates a counterfactual low level of debt to output. When that is the case, lenders become weary to finance a country that suffers mildly if it decides to default. Such is not the case in the current framework. Figure 6, characterizing numerically the average default event in the simulations, shows a fall of output of around 10%, accounted by a 8% slump in productivity and a 4% reduction in hours. However, due to lack of access to credit markets, the government increases endogenously the tax rate which depresses consumption by 10%. As the country advances into default, a debt restructuring negotiation leads to an agreement with international investors that implies repaying part of the overdue debt. To finance such repayment, the government has to raise taxes that distort labor even further, thus generating substantial losses for the household.

Similarly to the result regarding debt, reserves to output also matches the data with an average level of 9% in the ergodic distribution. In this model, reserves play an insurance role: if the country defaults without any reserves then consumption has to painfully adjust by bearing all the losses of default. However, since the probability of default is an endogenous choice for the country, that risk can be reduced by decreasing the debt it chooses to hold. Additionally, lower debt maps into lower interest rates. This channel would make reserves less important as an insurance instrument. Note that this is similar to the results in Alfaro and Kanczuk (2009): international reserves cannot be sustained in equilibrium[20]. The positive reserve holdings observed in the current simulations relate to the additional risk that the country cannot roll-over debt if hit by a sudden stop. In such scenario, the government finds it optimal to use reserves to repay debt, avoiding the negative impacts of a default. If the sudden shock carried further costs to the borrower, then reserves would play an even more prominent role as an insurance mechanism in the environment. In fact, such costs are present in this model as, to repay the outstanding debt, the government has to raise distortionary taxes with effects that are in everything similar to the ones described in the previous paragraph.

Other features of the data captured by this model refer to the negative correlation between the trade balance and output, and the default rate. Part of the reason why the model generates a larger default rate than in the data is accounted by the presence of the sudden stop shock. As shown in Roch and Uhlig (2018), adding a sudden stop shock with the characteristics presented in the model widens the borrowing risky region because, for certain shock realizations, the borrower is not able to roll over its debt. This effect increases the number of defaults in the economy. The result of a negative correlation between trade balance and output of $\mathbb{C}(TB/Y, y) = -0.55$ is also obtained in similar papers, for example, Aguiar and Gopinath (2006) or Yue (2010). Net capital outflows occur in recessions due to an increase in interest rates as the probability of default increases in recessions. This effect constrains borrowing endogenously.

Regarding the mean interest rate spread, the model is able to match the data target of 2.5%. This happens despite the presence of a positive recovery rate in the event of a default that becomes priced in new lending with lower spreads, the non-negative second term of the right hand side of equation (9). Matching the mean interest rate spread is an important feature of this model since it captures the government period-by-period cost of maintaining a portfolio of simultaneous positive reserves and debt.

One last relevant moment from table 5 is the negative correlation between output and the tax rate. To capture the fact that fiscal policies in emerging market economies are often pro-

[20]Several features in Alfaro and Kanczuk (2009) model explain their striking result: in addition to using a very simple model, without the features presented in the current paper, the authors assume a discount factor of 0.5 and a proportional output loss function.

cyclical, the model features an inelastic government expenditure with endogenous taxation. Cuadra, Sanchez, and Sapriza (2010) show that a similar environment can generate realistic pro-cyclical fiscal policies. As output declines, the government with constrained credit market access has to raise taxes to compensate the smaller base of taxation. The same idea is also used in this paper to generate distortions in production that are negatively correlated with output, thus generating more demand for reserves for insurance purposes. One current limitation of the current model is that government expenditure does not co-move with output. The main consequence of that assumption is an overestimation of negative correlation of the tax rate with output. However, as shown in Cuadra, Sanchez, and Sapriza (2010), making the government expenditure endogenous would still maintain a negative correlation of taxes and output–at the cost of adding on the computational burden–without qualitatively changing any of the mechanisms.

4.4 Macroeconomic dynamics around the average default episode

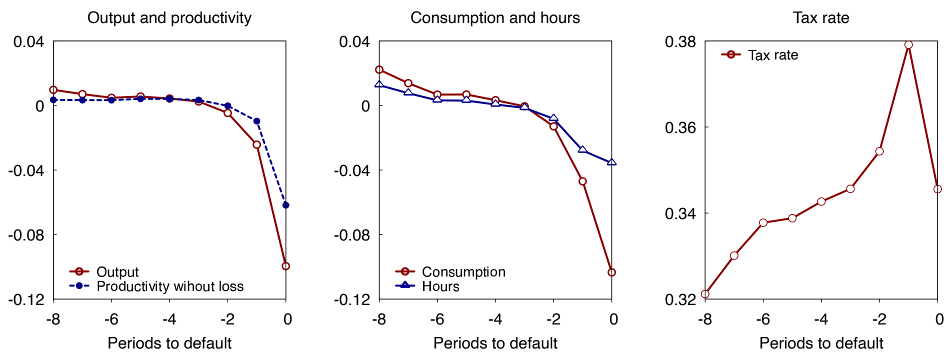
For all default episodes meeting the sampling criteria explained in the previous section, a time window of 9 periods is collected before a sovereign default episode occurs. Using this data sample, a period-by-period average of each variable of interest is computed and the date of default normalized to 0. These statistics are used to characterize, in figures 6 and 7, the macroeconomic dynamics that precede the average default event.

The left panel of figure 6 shows the dynamics of output and productivity around a default. The pattern of productivity is the standard in models of sovereign default. At $t = 0$, the level of productivity z becomes 6% below average.[21] Note that the depicted z does not include default costs. Once such exogenous costs, captured by function (22), are taken into account, the level of productivity falls to -8%. This translates into a fall of output relative to a trend of about 10% at the time of default due to a 4% decline in hours. Although large, the drop of output in period $t = 0$ already takes into account that, by defaulting, the sovereign releases resources to public and private consumption and so the tax pressure is reduced as shown in the right panel of the figure. In spite of a slight decrease at default, tax rates remain elevated which, in conjunction with productivity default losses, pushes consumption to decline by 10%. While direct tax evidence about labor taxes is difficult to collect for emerging market economies, using the methodology of Mendoza, Razin, and Tesar (1994), it is possible to construct series on average effective tax rates for OECD countries that experienced default or near-default crises in the past: Mexico, Portugal, Spain, or Greece. Inspection of these series confirms that taxes tend to rise around crisis periods. From equation (6), one can map tax rates in this model into Chari, Kehoe, and McGrattan (2007) definition of a labor wedge. Regarding financial crises, Tavares (2019) finds that labor wedges are major contributors for the observed movements of output. This implies that, in a more loose interpretation of the current

[21]This magnitude corresponds to a 1.5 standard deviation of the underlying shock as $stdev(\log z) = \sigma_z/\sqrt{1-\rho_z^2} =$ 3.9%.

environment, tax rates can be regarded as a set of unknown distortions in labor markets induced by the government.

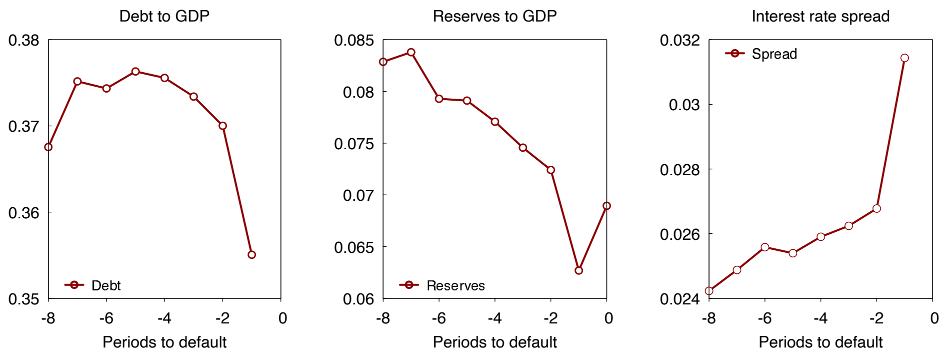
Figure 6: Dynamics preceding a default event: output, consumption, hours, and tax rates



Notes: This figure shows the model simulation averages for variables of interest in eight periods that precede a sovereign debt default. The two panels at the left show the percentage deviation relative to the average excluding periods of default or market exclusion. Additionally, the dashed line in the first panel shows the realization of productivity without the exogenous losses associated with equation (6). The third panel in the figure shows the average level tax rate in the simulations.

Also worth noting are the dynamics of reserves and debt depicted in figure 7. On average, up until period $t = -2$, the government maintains a relatively high level of debt that is being kept at a constant level of 37% to 38% with respect to output. However, at the same time, this corresponds to a period when reserves start falling from 8.5% to 7.0% of output. This implies that the government starts entering into a more risky position, as reflected with an increase of the interest rate spread from 2.4% to 2.7%. At $t = -1$, an additional negative shock in productivity generates a further deterioration of the interest rate schedule offered by the lenders. The government reacts by trying to decrease the current levels of debt and by depleting its stock of reserves. Nevertheless, the interest rate spread jumps to 3.1%. At this stage, given the fragile position of a government, a subsequent realization of a negative shock in productivity can throw the country into a default episode. On average, this occurs in the simulations when the next period's productivity falls an additional 5%, which takes place with a negative innovation that is two standard deviations below zero. When default finally arrives at $t = 0$, the sovereign keeps the level of reserves unchanged in anticipation of the debt restructuring that is resolved in the next period. Because of the fall in output, this generates an increase in reserves to GDP at default ($t = 0$) through a denominator effect.

Figure 7: Dynamics preceding a default event: debt, reserves, and interest rate spreads



Notes: This figure shows the model simulation averages for variables of interest in eight periods that precede a sovereign debt default. The first panel shows the average level of debt to GDP, the second the average level of reserves to GDP, and the third the average level of the interest rate spread. At the period of default $t = 0$, neither debt nor interest rate spreads are defined.

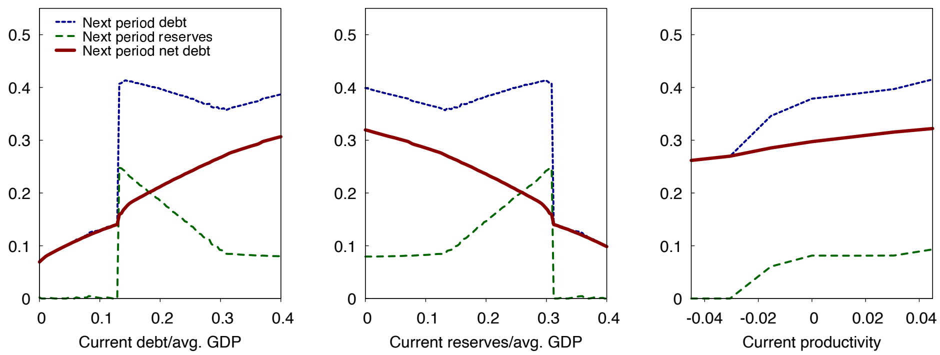
4.5 Understanding the mechanism

To better grasp the intuition[22] of the results presented in the previous sections, figure 8 depicts policy functions of the model when the economy is not hit by a sudden stop in the current period. That is, government choices for the next period debt level $D'(D, R, z)$, reserve holdings $R'(D, R, z)$, and net debt simply given by $D'(D, R, z) - R'(D, R, z)$.

The left panel of the figure shows these policies for different levels of current debt, fixing the current amount of reserves level and the productivity shock realization. The corresponding functions, $R'(D, \bar{R}, \bar{z})$ and $D'(D, \bar{R}, \bar{z})$, are further normalized with respect to the average output level generated in the simulations. Similarly, the middle and right panel show the same functions when current reserves or current productivity vary. The first observation is that reserves and debt are correlated and tend to move together. In particular, the right panel shows that gross capital flows, defined as $D' + R'$ are pro-cyclical and tend to collapse in recessions, consistent with empirical findings of Broner, Didier, Erce, and Schmukler (2013). Additionally, the two panels at the left show that the policy regarding net debt, represented by the solid line, is increasing with current debt but decreasing with current reserves. At very low levels of net debt, the sovereign chooses to carry zero reserves moving forward. Reserves provide no benefit to the policy maker when the probability of default is inexistent. However, as net debt increases, the probability of default switches to a positive number. At this point, reserves start providing insurance and the government loads immediately on both reserves and debt to take advantage of a low interest rate spread that emerges due to a

[22] A simple two-period model developed in section A.2 of the appendix can be useful to understand the intuition behind the full model.

Figure 8: Policy functions for next period debt, reserves, and net debt with respect to average output

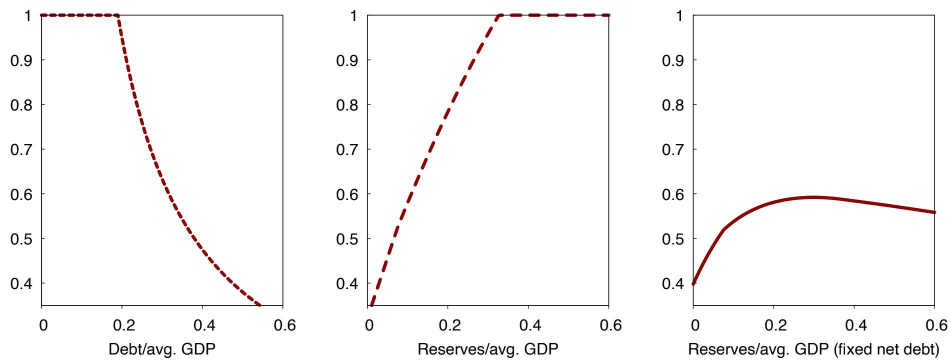


Notes: This figure plots government policy functions for debt holdings $D'(D, R, z)$, reserve holdings $R'(D, R, z)$ and net debt $D'(D, R, z) - R'(D, R, z)$. Reserves and productivity are fixed to the averages generated by the model simulations in the left panel; in the middle panel, instead, debt and productivity are fixed at their averages; and the right panel also fixes reserves and debt to their simulated averages. To ease the interpretation, the vertical axis in the three panels normalizes the policies with respect to the average output from the simulations. The same normalization is applied to the variables associated with the horizontal axes of the two panels at the left.

still small but positive probability of default. When net debt increases even further, the interest rate spread moves concomitantly, implying a larger cost of carrying positive amounts of debt and reserves. Closer to a default, and despite a larger interest rate spread, the sovereign chooses to maintain a positive amount of reserves to provide support for consumption if a bankruptcy and subsequent debt restructuring is triggered.

Figure 9 plots the equilibrium debt recovery rate schedule as a function of debt and reserves. Similar to the findings in Yue (2010), the debt recovery rate is decreasing with debt level. For a particular threshold, smaller amounts of debt generate a 100% recovery rate. A close inspection of problem (20) should clarify why the recovery rate schedule has that shape. In the bargaining problem, the settlement argument from φ is independent of the amount defaulted D. Then, the recovery rate schedule as a function of D becomes $0 \leq Rec(D) = \max\{1, \varphi/D\} \leq 1$. Intuitively, after defaulting, D becomes 'sunk' and agents only bargain about future values that do not depend on D. That is not the case with R since a marginal unit of reserves impacts differently the value of renegotiation and the outside option value for the sovereign (permanent exclusion). This means that different level of reserves implies different bargaining positions and, therefore, different debt reschedule outcomes. Notice that in financial autarky, the sovereign does not have any additional purpose other than to smooth consumption in subsequent periods. Given that the decision of restructuring prevails, by anticipating transfers the sovereign has an additional incentive to save on top of the previous consumption smoothing motive. With the marginal unit of reserves being more valuable under restructuring than under permanent autarky, the constraint of the negotiation problem is increasing in R. This argument justifies why the function of the right panel of the figure is increasing. Intuitively, reserves have more value for a government seeking credit market access than for a government staying in permanent autarky. The right panel of figure 9 illustrates this point by depicting a hump shape recovery rate schedule when reserves increase but net debt is kept constant (at the simulations average).

Figure 9: Recovery rate schedules for overdue defaulted debt



Notes: This figure plots recovery rate schedules as computed in the model. The left panel fixes the level of reserves to the average generated in the model simulations. The middle panel fixes the level of debt to the simulated average. And instead, the panel to the right fixes the net debt associated with the model simulations. That is, for a particular fixed amount of net debt $\overline{W}$, debt D increases with reserves R according to $R = D - \overline{W}$.

The shapes depicted in figure 9 are consistent with the evidence presented in section 2, figure 4. To better assess the consistency of the debt recovery rate schedule with the data, the same statistics presented in table 3 are constructed using using the artificial data simulated from the model. The results are presented in table 6 where, for convenience, part of table 3 is replicated. Overall the model performs well at capturing the correlations found in the data, that is, each additional percentage point of lagged reserves to GDP is associated with an average fall in the haircut of 1.7pp in the model, while the data generates a 1.5pp to 1.2pp fall.

Finally, the debt price schedules are plotted in Figure 10. The panel on the left shows that prices are decreasing with debt levels, as previously noted in the literature. More debt is associated with a higher default probability and lower recovery rate which translates into higher spreads. Notice that the debt price also improves with productivity, thus explaining why the correlation spread/output in table 5 is negative. This results from stronger incentives to default and lower recovery rates at low productivity levels. Additionally, both panels also show that a choice of more reserves is associated with smaller interest rate spreads. While keeping the debt choice level constant, larger reserves imply lower net debt in the following period, reducing the probability of a default and increasing

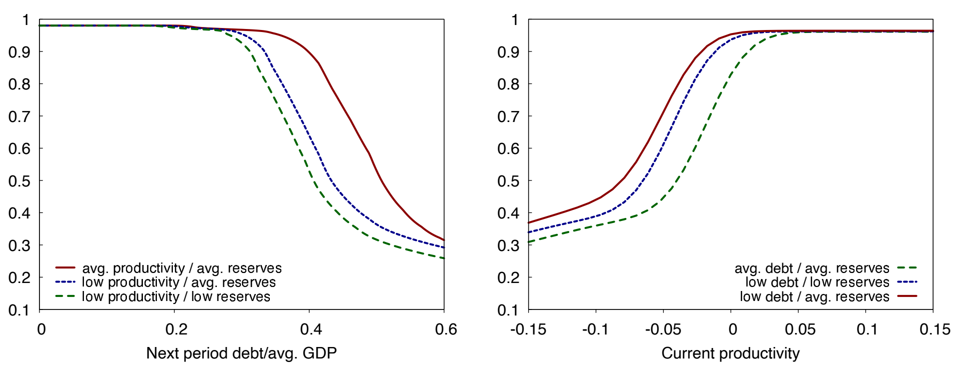
Table 6: Effects of international reserves on haircuts rates (% of overdue debt): data versus model

Dependent variable: Haircut rate at restructuring (%)	Benjamin and Wright (I)	Cruces and Trebesch (II)	Model simulations (III)
Lagged reserves to GDP (%)	-1.502	-1.214	-1.733
Lagged debt to GDP (%)	0.584	0.419	0.213
Lagged real GDP growth rate (%)	-0.302	0.208	-0.006

Notes: Columns (I) and (II) are taken directly from table 6. Column (III) shows the estimates of an OLS regression of the haircut (the counterpart of the recovery rate) against lagged reserves to GDP, debt to GDP, and the lagged GDP growth rate using all the artifical data generated in the simulation.

the debt recovery rate in the event of a bankruptcy. Both effects promote a positive correlation between spreads and reserves which is also consistent with the regression evidence presented in table 2 from section 2.

Figure 10: Price schedule for new debt as a function of current debt and productivity



Notes: This figure plots the sovereign debt price schedules characterized in equation (9). The left panel shows the schedule as a function of the future debt when both the current productivity and the choice of next period reserves are fixed. The right panel fixes the choices of debt and reserves, showing the schedule as a function of current level of productivity.

4.6 Sensitivity Analysis

The model presented in section 3 adds many non-standard features to the more traditional models of sovereign default. To help understand how these features change the results regarding the portfolio choice of debt and reserves, the baseline model is recomputed along some dimensions to highlight the mechanisms behind the results. For this exercise, the following alternative environments are evaluated:

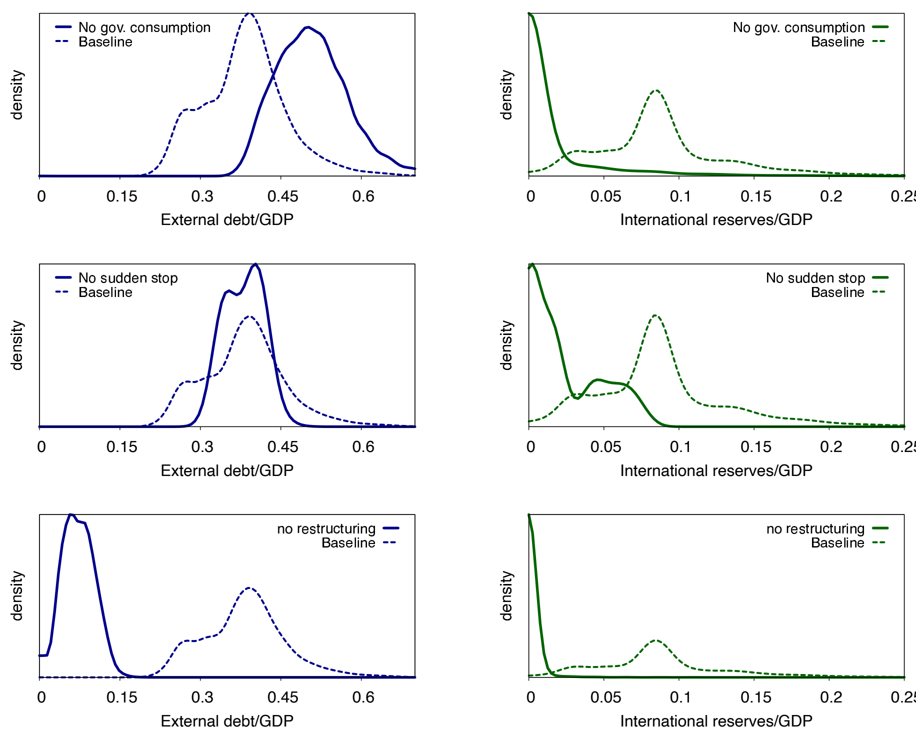
1. Assume that government expenditures are zero, with a simple implementation by just setting

$g = 0$.

2. Assume there are no sudden stops in the economy also implemented by setting $\omega = 0$.

3. Finally, assume that debt restructuring is shutdown in the economy. This can be implemented by increasing the parameter a by an amount large enough until the recovery rate schedule becomes zero at each point of the state space.

Figure 11: Simulated distributions for debt and reserves in different model specifications



Notes: This figure displays kernel density estimates for the distributions generated in the simulations for the different model specifications. Each panel compares the results from the alternative model against the baseline model. The description of each model specification can be found at the beginning of section 4.6.

To maintain comparability between the baseline and the alternative models, only the volatility and persistency of output are targeted by re-calibrating the parameters associated with the innovation of the productivity process, specifically σ_z and ρ_z. The newly computed alternative models are simulated to generate all the moments from table 5. Because the main interest is on the impact of debt and reserves accumulation, only the simulated distributions of these two variables are presented in figure 11 showing kernel density estimates of debt and reserves for a bandwidth choice varying between 0.005 and 0.03.[23] The panels at the left of the figure display the distributions of external

[23]The complete set of the simulated moments can be found in section A.4 of the appendix.

debt, while the ones at the right display the distributions of external reserves. Additionally, in each panel, the solid line captures the alternative model distribution and the dashed line the baseline model distribution.

The first two panels of the figure show that reducing the level of fiscal distortions through a lower average tax rate, imply a higher level of debt to GDP. At the same time reserves become less important for the sovereign. Reserves play an important role in the baseline economy as fiscal adjustments are costly and affect production. Eliminating such distortions reduces the need for reserves to 1% of GDP. The panels relative to sudden stops show that the presence of such shock is essential to generate realistic demand for reserves but do not completely eliminate their role. As long as the probability of default remains positive, the important movements of fiscal consolidation and debt restructuring requires some kind of insurance that can be provided by reserves. Finally, the last two panels show the most striking results from the exercise: shutting down renegotiation reduces abruptly the levels of debt and reserves holdings. Under this specification, the model resembles Alfaro and Kanczuk (2009) in the sense that reserves play no role in the optimal decision of the sovereign. This result confirms the findings in the previous sections showing that a substantial part of the cost associated with a default episode resides on debt restructuring. Table 7 summarizes the findings from this exercise.

Table 7: Simulation results of alternative specification of the baseline model

		Baseline	No gov. cons.	No sudden stop	No negotiation
Mean debt to GDP	$\mathbb{E}(D/Y)$	38%	51%	38%	7%
Mean reserves to GDP	$\mathbb{E}(R/Y)$	9%	1%	2%	0%

Notes: The entries in this table summarize the mean from the distributions depicted in figure 11.

5 Conclusion

This paper studies the extent to which reserves can have an important role as insurance instruments in a standard model that delivers realistic prediction regarding other dimensions of the data. Holding positive amounts of reserves is costly as current consumption is reduced when interest rates spreads between external debt and international reserves are positive. However, reserves can be beneficial if the economy experiences a financial crisis that forces consumption to adjust abruptly.

A model of equilibrium default augmented with reserve accumulation and sudden stop shocks is computed and simulated in order to evaluate if the benefits of holding reserves can compensate the costs. The model is calibrated to generate simulated moments that are similar to the ones observed for the Mexican economy. A positive and non-trivial average reserve to output can be generated while keeping quantitative realistic moments in dimensions such as debt to output, pro-cyclical trade balance, and co-movements of consumption and output. Reserves play an insurance role by allowing consumption to be smoothed if the economy defaults or is hit by a sudden stop shock. Default carries substantial adjustments when the government raises taxes in order to accommodate inelastic spendings and transfers of restructured debt back to lenders. These large penalties of default turn out to be fundamental in generating realistic demand for reserves: without distortionary taxation the demand for reserves drops from 9% of GDP to 1%. Without renegotiation, accumulation of reserves falls closer to zero. Moreover, endogenous renegotiation generates a debt recovery rate that is increasing with reserves and, in turn, affects the government's ex-ante incentive to default. In equilibrium, debt is priced to take into account the risk of default and the expected recovery rate. As such, the model predicts that interest rates and haircuts decrease with reserves, as observed in the data.

It should be noted that the results obtained abstain from many factors that, most likely, also influence the demand for reserves. Notably, this paper abstracts from the typical exchange rate management explanations (Obstfeld, Shambaugh, and Taylor, 2010). Recent developments in the literature have started to interact exchange-rate policy with optimal default (Na, Schmitt-Grohe, Uribe, and Yue, 2014), thus providing an analytical framework to integrate international reserves in the analysis. This is an issue to be addressed in future research.

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A Appendix

A.1 Data Sources

The evidence in section 2 uses the following variables:

Debt to GDP: from the *World Development Indicators,* as the sum of long-term external debt stocks, public and publicly guaranteed in current US$ (DT.DOD.DPPG.CD) with short-term external debt stocks in current US$ (DT.DOD.DSTC.CD) divided by the same year GDP also in current US$ (NY.GDP.MKTP.CD) at annual frequency.

Reserves to GDP: from the *World Development Indicators,* as total reserves (includes gold) in current US$ (FI.RES.TOTL.CD) divided by GDP in current US$ (NY.GDP.MKTP.CD) at annual frequency.

Spreads: computed as an arithmetic, market-capitalization-weighted average of bond spreads over US treasury bonds of comparable duration from data collected in *J.P.Morgan's Emerging Markets Bond Index Plus (*EMBI*)* at monthly frequency.

RGDP growth rate: from the *World Development Indicators,* as the annual percentage growth rate of GDP at market prices based on constant local currency (NY.GDP.MKTP.KD.ZG).

Revenues to GDP: from the IMF *World Economic Outlook,* as the general government revenue as a percent of GDP (GGR_NGDP) at annual frequency.

Expenditures to GDP: from the IMF *World Economic Outlook,* as the general government total expenditure as a percent of GDP (GGX_NGDP) at annual frequency.

Inflation: from the IMF *World Economic Outlook,* as the annual percentages of average consumer prices on year-by-year changes (PCPIPCH).

Openness: from the *World Development Indicators,* as the sum of exports of goods and services as % of GDP (NE.EXP.GNFS.ZS) with imports of goods and services as % of GDP (NE.IMP.GNFS.ZS) at annual frequency.

Contagion: constructed as the average of the EMBI spreads across all countries, but excluding the country of the observation, in a specific region (Latin America, Africa, Europe, Asia).

Rule of law: from the *Worldwide Governance Indicators,* this is an index constructed by the World Bank that measures perceptions of the extent to which agents have confidence in and abide by the rules of society, and in particular the quality of contract enforcement, the police and the courts, as well as the likelihood of crime and violence.

Urban population: from the *World Development Indicators,* as the urban population a percentage of total population (SP.URB.TOTL.IN.ZS).

Haircut: estimates taken from Benjamin and Wright (2009) on per dollar creditor losses is historical sovereign default episodes. Haircut estimates from Cruces and Trebesch (2013) are also used, made available and updated by Asonuma and Trebesch (2016), from where only defaults without a strict preemptive restructuring are selected.

Section 4 uses data from Mexico to construct business cycle statistics. Data is taken, at an annual frequency, from the OECD for 1960 to 2019. Debt and reserves uses data from the *World Development Indicators* also at an annual frequency. Moments are computed using de-trended series using an HP-filter. Consumption and income are expressed in logs. The default frequency is taken from historical default episodes identified by Reinhart and Rogoff (2009) in the last two centuries. Data for tax rates uses OECD *Revenue Statistics - Comparative tables* following Mendoza, Razin, and Tesar (1994) methodology to compute effective average tax rates on labor. In particular, the following variables are used:

GOS: Gross operating surplus and gross mixed income from OECD *National Accounts*

W: Compensation of employees from OECD *National Accounts*

1100: Income, profit and capital gains taxes of individuals from OECD *Revenue Statistics*

2000: Social security contributions from OECD *Revenue Statistics*

2200: Social security contributions of employers from OECD *Revenue Statistics*

3000: Payroll taxes from OECD *Revenue Statistics*

Using these variables, the two effective average tax rates are given by:

$$\tau^h = \frac{1100}{GOS + W}$$

$$\tau^l = \frac{\tau^h \cdot W + 2000 + 3000}{W + 2200}$$

Where τ^h is the personal income tax and $\tau^l$is the labor income tax, used to compute the correlation with output.

A.2 Two Period Model of Debt and Reserves

To better understand the intuition of the results presented in the main text a simple two-period model is introduced. Due to its designed simplicity, this model is not adequate to derive quantitative conclusions about the dynamics of the sovereign incentives to repay debt. However, different aspects of the role of reserves in a model allowing for debt default can be studied sequentially by assuming environments with different degrees of market completeness or commitment to repay debt.

The model environment involves an agent that lives for two periods and that is endowed with a certain income y_1 in the first period and an uncertain income y_2 in the second period that can only take two outcomes $y^h > y^l$. Assume that p is the probability of receiving y^h. Lifetime utility from consuming c_1 and c_2 is given by:

$$W = c_1 + \beta E u(c_2) = c_1 + \beta p \cdot u(c_2^h) + \beta(1-p) \cdot u(c_2^l) \tag{23}$$

where $u(c)$ is an utility function with $u' > 0$ and $u'' < 0$, and β is the discount coefficient. Note that the agent is risk neutral in the first period while risk averse in the second. This choice intends to capture, with a very simple specification, two distinct forces of sovereign default models: agents wish to bring consumption from the future into the present–captured by the coefficient β; agents wish to to smooth consumption emerging from uncertainty–captured by the uncertain endowment and the second period utility function.

Complete Markets

For the complete markets case, it is assumed that the agent can issue debt contracts with a risk-neutral external investor. Moreover, the agent can commit to repay her debts to the external investor. Under these assumptions, the agent budget constraint becomes:

$$c_1 = y_1 + q^h D^h + q^l D^l \tag{24}$$

$$c_2^i = y^i - D^i, \quad i = h, l \tag{25}$$

where q^h and q^l are prices for debt contracts if, respectively, the high or low income state of the world realize in period 2.

The typical solution for the maximization of (23), subject to (24) and (25) by choosing $\{D^h, D^l\}$ is given by:

$$q^h = \beta p u'(c_2^h)$$
$$q^l = \beta(1-p) u'(c_2^l)$$

And, if the borrower is small compared to the external investor, then:

$$\bar{q} = \beta u'(c_2) \tag{26}$$

where $\bar{q} = q^h + q^l$ is the risk-free debt price (or equivalently the external investor discount rate). Equations (26), (24) and (25) characterize the optimal allocation $\{c_1^*, c_2^*\}$ as function of $\beta, \bar{q}, y^h, y^l, p$.

Note that under complete markets consumption in the second period is certain even when income is not. This occurs because debt contracts are contingent.

Incomplete Markets with Full Commitment

Suppose now that the agent only has access to non contingent debt contracts. The budget set becomes:

$$c_1 = y_1 + \bar{q}D \tag{27}$$

$$c_2^i = y^i - D, \;\; i = h, l \tag{28}$$

Note that now debt D has to be re-payed whether the agent receives a high or low endowment. First order condition now implies

$$\bar{q} = \beta E u'(c_2^i) = \beta \left[p u'(c_2^h) + (1-p) u'(c_2^l) \right] \tag{29}$$

The solution of equations (27), (28) and (29), gives now an optimal allocation $\{c_1^*, c_2^{h*}, c_2^{l*}\}$ where $c_2^{h*} > c_2^{l*}$. That is, under incomplete markets the agent is not able to smooth consumption in the second period.

Incomplete Markets without Full Commitment

With an option to repudiate debt, an agent with debt D decides to default in period 2 if the associated consumption is larger than what would be the case under repayment. Assuming that the losses of default imply an income given by:

$$\tilde{y}(y) = \begin{cases} y^{d,h} & if \; y = y^h \\ y^{d,l} & otherwise \end{cases}$$

Then, for some level of debt D:

$$c^{def} > c^{rep} \Rightarrow c^{h,rep} = y^h - D > y^{d,h} = c^{h,rep} \tag{30}$$

$$c^{l,rep} = y^l - D < y^{d,l} = c^{l,rep} \tag{31}$$

This default choice implies that the price of one unit of uncontingent debt is $q = \bar{q}p$, so that the consumer budget constraint becomes:

$$c_1 = y_1 + qD \tag{32}$$
$$c_2^h = y^h - D \tag{33}$$
$$c_2^l = y^{d,l} \tag{34}$$

The first order condition and some algebra imply that $\bar{q} = \beta u'(c_2^h)$, that together with (32), (33) and (34) generate an allocation of consumption $\{c_1^*, c_2^{h*}, c_2^{l*}\}$ where $c_2^{h*} > c_2^{l*}$. Similar to the previous case, it is not necessary that consumption can be completely smoothed in the second period, irrespective of the contingency allowed by default.

Incomplete Markets with Reserves and without Full Commitment

Now the agent sees her contract choice expanded by reserves. The budget constraint is now given by:

$$c_1 = y_1 + qD - \bar{q}R$$
$$c_2^h = y^h + R - D$$
$$c_2^l = y^{d,l} + R$$

And the first order conditions by:

$$D: \quad q = \beta p u'(c_2^h) \Leftrightarrow \bar{q} = \beta u'(c_2^h) \tag{35}$$
$$R: \quad \bar{q} = \beta \left[p u'(c_2^h) + (1-p) u'(c_2^l) \right] \Rightarrow u'(c_2^h) = u'(c_2^l) \tag{36}$$

Implying an optimal allocation $\{c_1^*, c_2^*\}$. This result can be summarized in the following claim.

Claim 1. (Consumption smoothing) Under incomplete markets with reserves and without full commitment, $c_2^h = c_2^l = c_2$ is a solution for a given $\tilde{y}(y)$ and β.

Proof. For a given D choose $\tilde{y}(y)$ such that (30) and (31) hold. Given (35), such D can be supported for a specific β. The rest of the proof follows directly from the first order conditions. □

This result shows that reserves and the ability to default allows the agent to perfectly smooth consumption in the second period. In this sense, reserves are useful to complete the market.

Claim 2. (Demand for reserves and loss of default) Under incomplete markets with reserves and without full commitment $\partial R / \partial y^{l,d} < 0$ for a given $\tilde{y}(y)$ and β

Proof. This result follows from applying the implicit function theorem to (36) and claim 1:

$$\begin{aligned} & \bar{q} && = \beta\left[pu'(c_2^h) + (1-p)u'(c_2^l)\right] \\ \Leftrightarrow\quad & \bar{q} && = \beta\left[\bar{q}/\beta + (1-p)u'(y^{d,l} + R)\right] \\ \Rightarrow\quad & \tfrac{\partial R}{\partial y^{d,l}} && = -1 < 0 \end{aligned}$$

□

The result from claim 2 says that the demand for reserves is increasing with the output loss. This result provides the intuition why the full model adds counter-cyclical distortionary taxation: increasing taxes when the economy is in default further depresses the economy thus increasing the usefulness of reserves in such events.

Reserves and Renegotiation

Consider that if the agent defaults in the second period, lenders make a one-shot offer of debt restructuring by granting market access to the agent. That option grants the following utility to the lender:

$$W^d = u(c_2) + A$$

where A is meant to capture in this simple framework all future benefits of regaining access to international credit markets. The lender makes an offer of a debt restructuring $\tilde{D}$ such that the agent is indifferent in accepting and rejecting the offer, that is:

$$u(y+R) = u(y+R-\tilde{D}) + A$$

That equation implicitly defines a recovery schedule $\tilde{D}(y,R,A) \leq D$ where D is the defaulted debt. The following result follows:

Claim 3. (Recovery rate is increasing with reserves) $\partial\left(\tilde{D}/D\right)/\partial R \geq 0$

Proof. This follows immediately from the fact that $u' < 0$ and $y + R \geq y + R - \tilde{D}$. □

This result indicates that an agent is willing to transfer larger amounts of reserves to lenders if there are gains of regaining market access.

A.3 Numerical computation

The computational algorithm is coded in Fortran and is similar to the one used in Arellano (2008) or Yue (2010). Some recommendations from Hatchondo et al. (2010) on how to solve numerically sovereign debt models are followed. For error tolerance ϵ_v and ϵ_φ, the algorithm steps follow the below sequence:

1. Discretize space of D, R and the shock process z
2. Solve first for the value of permanent autarky $v^{aut}(R,z)$ using value function iteration:
   (a) guess $v^{aut,0}(R,z)$ and generate tensor basis splines of degree 2 (linear) over the two dimensions R and z
   (b) using $v^{aut,0}(R,z)$ solve for the maximization (17) to get $v^{aut,1}(R,z)$, using a non-linear equation solver to determine the labor equilibrium in (18) and (19); and first a grid search method over the space of R and then a numerical optimizer over continuous choices using the approximations from the b-splines and the first optimum as a guess.
   (c) evaluate $\left||v^{aut,1}(R,z) - v^{aut,0}(R,z)\right|$; if it's larger than ϵ_v iterate on (a) using $v^{aut,0}(R,z) \coloneqq v^{aut,1}(R,z)$ until convergence
3. Guess the the recovery function $\varphi^0(D,R,z)$, and the value function $v^{rep,0}(D,R,z,s)$
4. Use $\varphi^0(D,R,z)$ to update the value function to $v^{rep,1}(D,R,z,s)$ using a grid search method over the space of (D,R); generate tensor basis splines of degree 2 (linear) over the three dimensions D, R, and z:
   (a) labor equilibrium in (11) and (12) is obtained using a non-linear equation solver
   (b) $v^{def}(D,R,z)$ by maximizing (13) using $\varphi^0(D,R,z)$ and $v^{rep,0}(D,R,z,s)$, first using a grid search method over the space of D and R and then a numerical optimizer over continuous choices using the approximations from the b-splines and the first optimum as a guess.
   (c) the debt price function is computed from the definition (9) using the functions $\varphi^0(D,R,z)$, $v^{rep,0}(D,R,z,s)$ and $v^{def}(D,R,z)$
5. Evaluate $\left||v^{rep,1}(D,R,z,s) - v^{rep,0}(D,R,z,s)\right|$; if it's larger than ϵ_v iterate on (4) using $v^{rep,0}(D,R,z,s) \coloneqq v^{rep,1}(D,R,z,s)$ until convergence
6. Using $v^{def,1}(D,R,z)$, compute $\varphi^1(D,R,z)$ by solving (20) using a non-linear equation solver
7. Evaluate $\left|\varphi^1(D,R,z) - \varphi^0(D,R,z)\right|$; if it's larger than ϵ_v iterate on (3) using $\varphi^0(D,R,z) \coloneqq \varphi^1(D,R,z)$ until convergence.

In the benchmark model, the state space of D and R is $[0; 0.65]$ with 201 equally spaced grid-points, that is, both debt and reserves share the same state space. The productivity process $\log z$ is discretized with 35 grid points using the method proposed in Tauchen (1986) with bounds given by a margin of 5.5 unconditional standard deviations with respect to the mean. The maximum error allowed is $\epsilon_\nu = 10^{-6}$. The non-linear equation solver uses the Fortran algorithm BRENT from Richard Brent, the maximizer uses the Fortran algorithm NEWUOA from Michael J. D. Powell, and evaluation of expectations uses a Gauss–Legendre quadrature rule.

A.4 Simulation results for alternative specifications of the model

This section shows the simulation results from using alternative specifications of the model using the parameter values from table 4.

Table 8: Business cycle statistics for the baseline model and alternatives

	Data	Baseline	No gov. cons.	No sudd. stop	No reneg.
$\mathbb{E}\left(D/Y\right)$	37%	38%	51%	38%	7%
$\mathbb{E}\left(i^{spread}\right)$	2.5%	2.5%	2.9%	0.5%	4.9%
$\mathbb{SD}\left(TB/Y\right)$	1.9%	2.4%	3.8%	2.3%	2.3%
$\mathbb{SD}\left(y\right)$	3.2%	3.9%	3.9%	3.9%	3.9%
$\mathbb{AC}\left(TB/Y\right)$	0.63	0.67	0.67	0.67	0.67
$\mathbb{E}\left(D^{recov}/D\right)$	55%	52%	45%	33%	0%
$\mathbb{E}\left(G/Y\right)$	15%	16%	0%	16%	16%
$\mathbb{E}\left(R/Y\right)$	8%	9%	1%	2%	0%
$\mathbb{SD}\left(i^{spread}\right)$	0.9%	0.7%	0.6%	0.5%	6.3%
$\mathbb{SD}\left(c\right)/\mathbb{SD}\left(y\right)$	1.4	1.5	1.5	1.5	1.5
$\mathbb{C}\left(c,y\right)$	0.93	0.96	0.93	0.97	0.96
$\mathbb{C}\left(i^{spread},y\right)$	-0.39	-0.46	-0.49	-0.20	-0.75
$\mathbb{C}\left(TB/Y,y\right)$	-0.60	-0.55	-0.55	-0.54	-0.49
$\mathbb{C}\left(\tau,y\right)$	-0.49	-0.74	-0.55	-0.74	-0.72
$\mathbb{E}(\tilde{Z})$	2-5%	4.6%	4.4%	0.7%	4.4%