Competition for Managers and the Rise of the Skill Premium*

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1 Introduction

Managerial occupations represent a significant and expanding segment of the US labor force. At the same time, good managerial practices enhance production efficiency. Competition among firms to hire managers' services influences their remunerations depending on the technological contribution of a manager to efficiency and on the scarcity of these services in the market. Moreover, since the improvement in the efficiency of production also changes firms' demand for other factors of production, the general compensation of other types of labor, particularly high-skill workers, also change in a market economy. Given the extensive and expanding body of research on the rise of wage inequality in the US economy[1], this paper adds to the literature by analyzing how the increase in managerial compensation can be accommodated with an expansion of their services and how these contribute to the increase in the skill-premium.

Answering this question requires bridging two related literatures. On one hand, studies such as Ichniowski et al. (1997), Syverson (2011),or Bloom et al. (2012) show that managerial practices influence firms productivity. This literature highlights that the proliferation of effective managerial practices and the increased delegation of CEO tasks to middle management positions are associated associated with heightened firm sales, value added, and employee compensation. On the other hand, economist have been documenting a raise in the skill premium, linked with growing wage inequality between low-skill and high-skill workers, even amid an increased relative supply of the latter. Typical explanations for this phenomenon rely on models that add various forms of skill-biased technical change that stimulate higher demand for high-skill labor, thereby widening the wage gap. Several authors have proposed different forces that may trigger this type of technological shift in favor of high-skill labor, for example, exogenous technological growth, capital-skill complementaries, or structural change (Tinbergen, 1974, Krusell et al., 2000, or Buera et al., 2021). In the present study, we partially endogenize this skill-biased technical change by allowing managers to enhance firms' productivity. We show that a simple model that accommodates this role for managers can account for most of the increase in the high-skill premium as observed in the data, while simultaneously generating a trajectory for managerial compensation in the US economy.

Using census data from the American Community Survey (ACS), we start by documenting an expansion of the relative importance of managerial occupations in the US labor force. Between 1950 and 2019 we observed an increase of occupations related to managers from 4% to about 20%. At the same time, the managers' premium, measured as the relative hourly wage of manager occupation relative to the wage of low-skill non-college educated workers increased from 39% to 100%. To inspect how the expansion in managerial positions also affected wages of high-skill workers we augment the typical Tinbergen (1974) regressions that project the log wage premium of high-skill workers (non-managers) against their relative supply and a time trend by also including the relative supply

[1]A non-exhaustive list include Tinbergen (1974), Katz and Murphy (1992), Krusell et al. (2000), Acemoglu and Autor (2011), Carneiro and Lee (2011), or Card et al. (2018) among others.

of managers. We find results consistent with the empirical literature that accounts the increase in the skill-premium to both supply factors (the relative abundance of high-skill workers) and demand factors (the skill-biased technical change captured by the time trend). However, when including the relative supply of managers in the regressions we find positive and significant coefficients and a diminished estimated role for the time trend. These results provide suggestive evidence consistent with a non-negligible role of managers at accounting for the observed patterns of the skill premium.

To study such relationship, we introduce a simple model where firms compete in a monopolistic environment. Production accrues from using three different labor inputs: low-skill workers, high-skill workers, and managers. Low-skill and high-skill workers are hired in a competitive input market. Instead, managers are hired in an initial stage where firms compete for their services by biding compensation offers. Because in our environment managers increase the relative efficiency of high-skill labor, firms are willing to offer transfers to the managers that equate the profit opportunity cost of not hiring managers. In the model characterization we show that, in equilibrium, this compensation relates not just related to the increased technological efficiency provided by the managerial services, but also with the wages of high-skill workers. We find that a increase in the aggregate supply of managers in a economy can lead to a higher managerial premium, with this effect reverting when the level of managers in the economy is sufficiently large to induce a offsetting effect in the wages of the high-skill workers (that also increase with the supply of managers).

Using the model structure we then perform a quantification of the mechanisms highlighted relying on a standard calibration strategy. To do this, we input the model with the observed sequence of relative supply of low-skill, high-skill, and managerial labor as observed in the data. Additionally, we add an additional auxiliary exogenous skill-biased technical change free-variable that allow us to match exactly the observed path of the high-skill premium. In this exercise, we find that the model mimics the pattern of the managers' premium displayed in the US economy between 1950 and 2019. At the same time, albeit present, we find a diminished role of the exogenous skill-biased technical change. This indicates that the inclusion of managers in an, otherwise, standard model of income distribution in a economy can account for a large part of the observed increase in the skill-premium.

The rest of the paper is organized as follows. Section 2 provides empirical motivation for the mechanisms highlighted in the model that is introduced in section 3. Next, in section 4, we use a calibrated version of the simple model to assess the quantitative validity of the role of managers at explaining the skill premium. We provide conclusions in section 5.

2 Empirical motivation

In this section, we outline the main trends of the wage distribution and occupational choice in the US labor market since the 1950s, with a particular focus on workers with a high school education or less compared to those with a college education or more. Consistent with previous studies[2], we observe an increase in the wage premium of college-educated workers despite an increase in its relative supply. This pattern is particularly pronounced when we restrict our sample to college-educated workers in managerial and related occupations. Moreover, we examine the cross-sectional variation in these trends and document that sectors with larger increases in the relative supply of managers correspond to sectors with faster growth in the wage premium for non-manager, college-educated workers.

2.1 Data

To document the facts presented we rely on the US Census samples and the American Community Survey (ACS).[3] The US Census data encompasses 1% samples of the US population for the years 1950, 1960, 1970, 1980, and 1990. For years following 2000, we use the ACS, an annual survey conducted by the Census Bureau. The ACS has similar questions to the Census and provides a 1% random sample of the entire U.S. population. To maintain consistency with the US Census, we include ACS data from 2000, 2010, and 2019 in our analysis.[4] One key advantage of the data provided by the U.S. Census/ACS is its substantially larger sample sizes compared to alternative datasets such as the Current Population Survey (CPS), typically used to measure the skill premium. This allows for a detailed analysis of the changes in wages and occupational employment across different sectors while controlling for fine-grained individual characteristics.

Each observation contains information on an individual's demographics, education, occupation, sector of activity, wage income, and work hours. We limit our sample to employed workers aged between 25 and 60 years old at the time of the survey. Individuals are categorized into five educational groups (less than high school, high school, some college, college, and more than college), four groups of potential experience (9 or less years, 10 to 19 years, 20 to 29 years, and more than 29 years), and by sex (male, or female). We also distinguish observations between managerial and non-managerial occupations using a harmonized coding scheme based on the Census Bureau's 2010 ACS as provided by IPUMS. Occupations associated with 'management, business, science, and arts', 'business operations specialists', and 'financial specialists' are included in our broad category of managerial roles.

The data is further restricted to full-time workers, who report more than 40 weeks of work over the previous year. Observations with hourly wages that are 50% below the federal minimum wage or with a weekly wage smaller than $50, adjusted for inflation using the Consumer Price Index (CPI) with 1982 as the base year, are also dropped. Workers are categorized into sectors of activity

[2] See, for example, Tinbergen (1974), Katz and Murphy (1992), Krusell et al. (2000), Autor et al. (2008), Carneiro and Lee (2011), or Hoffmann et al. (2020).

[3] Data from both the US Census and the ACS can be retrieved from the IPUMS website (https://usa.ipums.org).

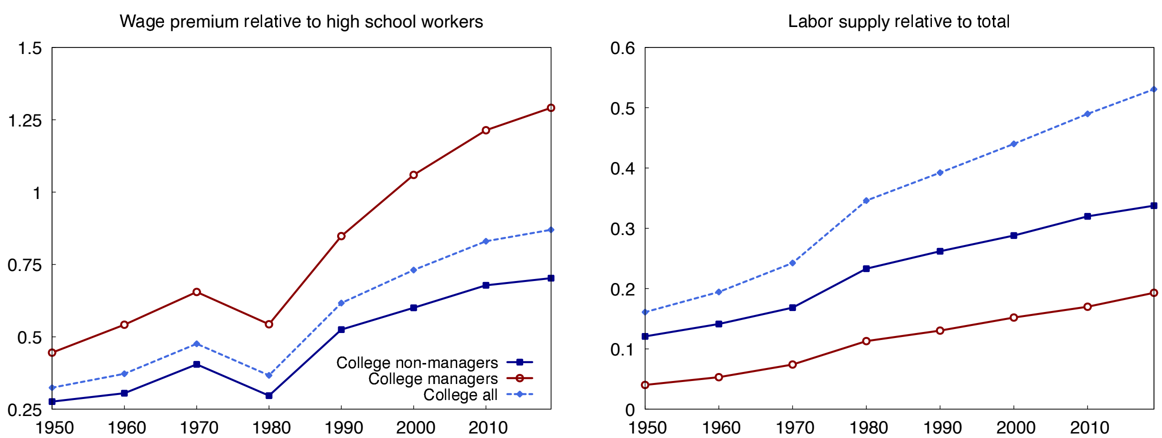
[4] The inclusion of the ACS survey of 2019 instead of the one from 2020 is related with the impact of the covid-19 epidemic on the collection and data quality that induced the Census Bureau to use experimental weights on the observations.

using the 1990 Census Bureau industrial classification scheme, which allows for sector harmonization across years. This enables us to divide our data into 12 sectors.[5] It is with this individualized data on hourly wages and total hours worked that we compute relative wages and relative labor supply across various educational and occupational groups in the subsequent analysis.

2.2 Managers intensity and the college education wage premium

As our objective is to examine trends in the skill premium and managerial compensation, we first illustrate wage dynamics and work hours over the past seven decades in the US economy. We first define 'skilled' and 'unskilled' workers as those with completed college education or higher, and those with incomplete college education or less, respectively. Within the 'skilled' category, we further distinguish between 'managerial' and 'non-managerial' occupations based on our classification. For each group, we compute total yearly hours as the product of reported weeks of work over the year with the typical number of hours worked in a week. Hourly wages are determined by dividing total annual labor income by the total hours worked.

Figure 1: Composition-adjusted wages and relative labor supply across education and occupations



Source: US Census samples and the American Community Survey.

To measure average wages within groups, we adopt the methodology used by Acemoglu and Autor (2011), computing composition-adjusted wages in the following manner. First, for each year, we project log real wages onto dummy variables that capture our previously defined groups, specifically, two sexes, five educational groups, four potential experience brackets, and two occupations. Second, we calculate mean wages for broader groups (managers, high, and low skill workers), holding constant the relative employment shares of our 40 labor groups across all sample years. This method ensures that changes in average wages are not the result of shifts within the narrow groups'

[5]The appendix A.1 lists the included sectors and provides additional details about the data cleaning.

composition of sex, education, experience, or occupation. Figure 1 plots the trend of composition-adjusted wages for college workers relative to high school workers from 1950 to 2019, as well as the relative supply of hours.

The figure illustrates trends in the skill premium and relative labor supply similar to those documented in other studies.[6] Specifically, we observe an increase in the wage gap from 32% in 1950 to 83% in the 2010, and 87% in 2019. At the same time, the supply of college educated relative to the total supply of workers increased from 16% in 1950 to 49% and 53% in 2010 and 2019, respectively. This pattern becomes even more pronounced when we differentiate college-educated workers into managerial and non-managerial occupations. Between 1950 and 2019, the wage premium of managers soared from 45% to 129%, while the relative labor supply of college-educated managers rose from 4% to 19%.

Table 1: Across-sectoral trends in composition-adjusted wages and relative labor supply (1950/2019)

		Managers				Non-managers			
		%wage gap		%labor supply		%wage gap		%labor supply	
	Sector	1950	2019	1950	2019	1950	2019	1950	2019
1	Agriculture and fisheries	71	79	3	11	78	62	1	17
2	Mining	41	130	3	18	31	80	9	19
3	Construction	32	84	3	12	16	30	5	8
4	Manufacturing	72	168	4	22	32	90	8	24
5	Transportation and comm.	51	112	2	13	21	58	5	22
7	Retail and wholesale trade	50	107	4	9	25	50	4	22
8	Finance and real estate	47	109	9	43	16	68	13	27
9	Business and repair serv.	63	160	5	26	31	93	9	35
10	Personal services	45	119	3	14	12	39	3	18
11	Entertainment	30	84	5	20	24	40	8	31
12	Professional serv.	68	122	4	18	39	76	56	54
13	Public administration	54	68	4	17	28	44	16	41
0	All sectors	45	129	4	19	28	70	12	34

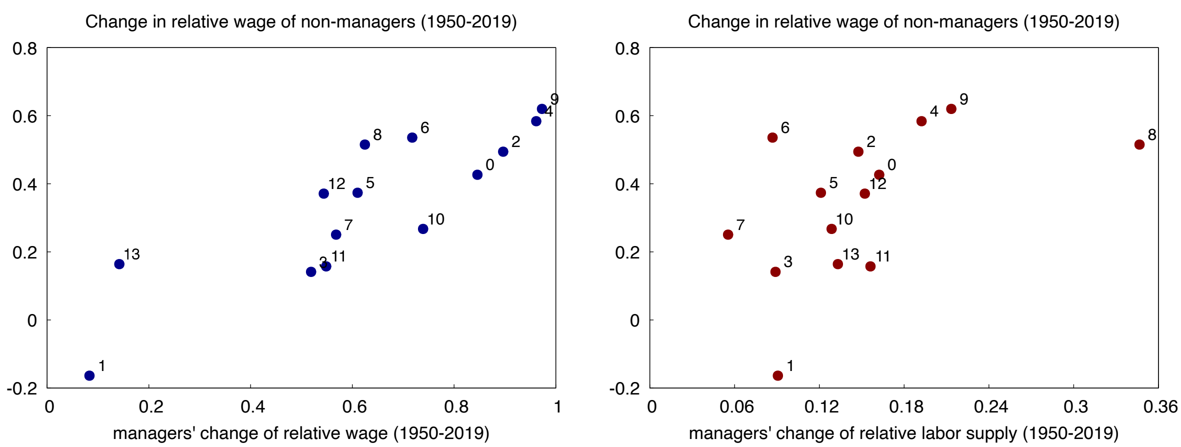
Source: US Census samples and the American Community Survey.

Similar results emerge when we measure composition-adjusted wages within each broad sector in the sample. Table 1 shows the evolution of these measures between 1950 and 2019. We observe common general trends across sectors where both the wage premium and the relative supply increase for managers and non-managers. However, these increases vary considerably across

[6]For example, Autor, Katz, and Kearney (2008), Acemoglu and Autor (2011), or Autor (2022) also document trends in the skill premium and relative labor supply with similar patterns and magnitudes to the ones presented in figure 1.

sectors. For instance, the increment of managerial employment ranges from 5 percentage points (retail sector) to 34 percentage points (finance), while the premium increase ranges from 8 percentage points (agriculture) to 96 percentage points (manufacturing). A similar pattern is evident for college-educated non-managerial workers, albeit with less intensity. Notably in figure 2, we observe a clear cross-sectoral positive correlation between wages of college-educated non-manager' and both managers' wages or managers' relative labor supply. This suggests a potential interaction between college-educated workers wages and the intensity of managers utilization and their respective compensation.

Figure 2: Correlation between wages and labor supply across sectors of activity



Source: US Census samples and the American Community Survey.

To better understand the relationship between the observed patterns of the college-educated non-manager premium, we adopt the standard analysis from Tinbergen (1974).[7] This analysis is inspired by a canonical model of the labor market, in which the evolution of the college premium is explained through the effect of skill-biased technical change (demand factors) and the availability of high-skill to low-skill labor (supply factors). The forces associated the the demand and supply factors are usually measured through the recourse of linear regressions where the log wage premium for college-educated workers is regressed against a time trend and their relative supply. In this paper, given our interest on role of managers, we augment the standard estimation equation by including the relative supply of managers. Our main regression specification estimates:

$$w_{jt}^{college} = \alpha + \beta_0 \times t + \beta_1 \times h_{jt}^{college} + \eta \times h_{jt}^{managers} + \gamma_j + \epsilon_{jt}$$

where $w_{jt}^{college}$ is the logarithm of the ratio of the average wage of college-educated (non-manager)

[7]This framework has been applied by many authors studying the college premium, for example, Katz and Murphy (1992), Card and Lemieux (2001), or Acemoglu and Autor (2011).

workers to high-school workers at time t in sector j, $h_{jt}^{college}$ the logarithm of of the relative supply of college (non-managers) hours, and $h_{jt}^{managers}$ the relative supply of mangers' hours. Additionally, we include a common time trend variable t, and a sectoral fixed effect γ_j. The residual of the regression is captured in ϵ_{jt}.

Table 2: Regression models for the college non-manager wage premium between 1950 and 2019

Dependent variable: log relative wage college to high school	(I)	(II)	(III)
Regressors:			
relative employment of college workers	.074**	-.079**	-.079**
	(0.25)	(.034)	(.033)
relative employment of managers			.575**
			(.238)
time trend		.0053***	.0043***
		(.0011)	(.0012)
sectoral fixed effects	Yes	Yes	Yes
Observations	96	96	96
R-squared	0.19	0.61	0.63

Source: US Census samples and the American Community Survey.

Table 2 displays the estimation results, which align well with the canonical model. Specifically, when we incorporate a time trend into the regression, the coefficient on the relative supply of college hours turns negative. A direct interpretation of the coefficients suggests that the trend variable captures demand effects via skill-biased technical change (with a positively estimated sign), while the relative employment of college-educated workers captures supply effects (with a negatively estimated sign). Augmenting the regression to include a relative employment of managers independent variable, does not change the pattern of correlations for the time trend or for the relative employment of college educated workers. However, the estimated sign for this new variable is positive and significant, suggesting that a higher prevalence of managers in a sector may increase demand for college-educated workers implying higher wages. Consistent with this interpretation is the 20% decrease in the estimated coefficient for the time trend between regression (II) and (III): after controlling for the relative employment of managers, the time trend effect on the relative wage of college educated workers becomes less important.

2.3 Summary

In this section we utilized ACS data to document a significant increase in the wage premium of high-skill labor in the US over the last 70 years. This increase in the wage premium is especially pronounced in managerial occupations. Concurrently, the relative supply of both managers and high-skill labor has also increased. These data trends are consistent with standard neoclassical theory, provided we account for the role of skill-biased technological demand factors. This is corroborated by our high-skill wage regression results, which show a negative coefficient for the supply of high-skill labor (indicating a negative supply effect) and a positive coefficient for a time trend (indicating a positive skill-bias technical change effect). However, when we augment these regressions to include the effect of the relative supply of managers, we uncover a positive impact of managers on high-skill wages and a reduced importance of the time trend. We find this evidence suggestive of a role of managers at explaining part of the dynamics of the skill-premium in the US.

3 A simple model

Given the empirical results documented in the previous section, we propose a simple model that ilustrates how changes in the supply of different type of workers (managers, high-skill, and low-skill) contribute to wage inequality. In our framework, we portray managers as agents who can directly influence firm productivity and, consequently, profits. Under some conditions, competition among firms for the limited availability of managers implies that managers' salaries can increase along with their supply, thereby also elevating the income of high-skill workers.

We analyze a general equilibrium economy with a monopolistically competitive product market wherein firms employ managers, high-skill, and low-skill workers to produce imperfectly substitutable goods. The economy comprises two classes of agents—a continuum of identical households of measure 1, and two firms (or sectors). Households consume two goods that are imperfect substitutes and supply three types of workers—namely, managers, high-skill workers and low-skill workers who are in fixed supplies, M, H and L, respectively. Each firm produces one good by employing all three types of workers. Think of the workforce of the economy consisting of 'college graduates' and 'high school graduates'. The high school graduates are designated as 'low-skill workers', whereas the college graduates comprise 'managers' and 'high-skill workers'. Unlike high- and low-skill workers, whom the firms hire in a competitive labor market, managers are hired in firms through a bidding process.

Preferences and technology. A representative household derives utility from the consumption of the two goods that are imperfect substitutes. The utility function is given by

$$U(x_1, x_2) \equiv \left(x_1^{\frac{\sigma-1}{\sigma}} + x_2^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

with $\sigma > 1$ representing the elasticity of substitution between the two goods. In a monopolistically competitive product market, σ determines the market power of the firms—the higher the product substitutability, the lower is the market power firms enjoy.

Firms (potentially) differ in their production technology which are given by

$$y_i = f(h_i,\, l_i;\, z_i) \equiv A\left(\alpha(z_i h_i)^{\frac{\zeta-1}{\zeta}} + (1-\alpha) l_i^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}}, \tag{2}$$

Parameters A, α and $\zeta \leq \sigma$ are technology parameters: A represents a Hicks-neutral technological change, $\alpha \in (0,\, 1)$ is the factor intensity associated with high-skill workers, and $\zeta > 1$ is the elasticity of input substitution between high- and low-skill workers. The parameter z_i represents the firm-specific skill-biased technological change (SBTC). Our framework endogenizes z_i. In particular, firm i's SBTC is given by

$$z_i = z(m_i) \equiv z_0 + m_i^{\gamma}, \quad z_0,\, \gamma > 0. \tag{3}$$

The firm-specific SBTC depends on some firm attribute, z_0 which is same across both firms (e.g., firm size, baseline input) and the number of managers employed in firm i, m_i. Firm i produces good i by using employing managers, high-skill workers (in quantity h_i) and low-skill workers (in quantity l_i).

The timing of events. The economy lasts for two subperiods, $t = 1,\, 2$. At $t = 1$, firms hire managers from the pool of M managers. We assume that managers are not hired in a competitive labor market, rather through a bidding process. Each firm i 'bids' for the managers by offering per-manager salary w_m^i to employ m_i managers in the firm. Once m_i managers are employed in firm $i = 1,\, 2$, the SBTC, $z(m_i)$ becomes common knowledge. At date 2, each firm i hires high- and low-skill workers at competitive wages $(w_h,\, w_l)$, and carry out the production of good i. At the same time, the representative household submits the demand for each good by taking its prices as given. Finally, firms set prices by maximizing profits.

3.1 Equilibrium

The general equilibrium of our economy is solved sequentially in the two subperiods. We first determine the salary offers for each manager by the firms 1 and 2, $(r_1,\, r_2)$, and the allocation of managers across firms, $(m_1,\, m_2)$ such that $m_1 + m_2 = M$. The equilibrium of the stage 2, given $(m_1,\, m_2)$, is a standard Walrasian equilibrium of the economy that comprises a monopolistically competitive product market where goods 1 and 2 are traded at prices $(p_1,\, p_2)$, and a competitive labor markets where firms hire high-skill and low-skill workers at wages $(w_h,\, w_l)$. We normalize w_l, the low-skill wage, to 1, and write $w_h \equiv w$.

In stage 2, the representative household submits demands for both goods by maximizing utility in (1), taking the prices $(p_1,\, p_2)$ as given, i.e.,

$$(x_1(p_1,\, p_2),\, x_2(p_1,\, p_2)) \equiv \underset{\{x_1,\, x_2\}}{\text{argmax}}\ \{U(x_1,\, x_2) \mid \text{subject to } p_1 x_1 + p_2 x_2 = I\}, \tag{4}$$

where $I > 0$ is the household income. Firm profits and the aggregate worker (managerial, high- and low-skill) incomes accrue to the households, and hence, the household income, I is the sum of the profits of the two firms, and aggregate worker incomes.

Firms optimize in two stages. First, each firm i employs high- and low-skill workers by minimizing cost, taking w and the production technology in (2) as given, i.e., firm i solves

$$C_i(y_i) \equiv \min_{\{h_i, l_i\}} \{wh_i + l_i \mid \text{subject to } y_i = f(h_i\, l_i;\, m_i)\}. \tag{5}$$

Next, firms compete in the product market. The market-clearing condition for each good implies $x_i(p_1,\, p_2) = y_i(p_1,\, p_2)$ for $i = 1,\, 2$. Thus, each firm i sets price p_i to maximize profit, i.e.,

$$\tilde{\pi}_i \equiv \max_{p_i} p_i y_i(p_1,\, p_2) - C_i(y_i(p_1,\, p_2)). \tag{6}$$

We solve the model by backward induction.

3.2 Analysis of the equilibrium

Hoseholds. The economy has 2 goods, 1 and 2. A representative consumer has utility function $U(x_1,\, x_2)$. We assume The consumer, given the prices $(p_1,\, p_2)$, maximizes the above utility function subject to the budget constraint

$$p_1 x_1 + p_2 x_2 = I,$$

where I is consumer's income. Optimality implies

$$\frac{x_1}{x_2} = \left(\frac{p_2}{p_1}\right)^{\sigma}.$$

Substituting the above into the budget constraint one can derive the following demand functions:

$$x_1 \equiv x_1(p_1,\, p_2,\, I) = \frac{I}{P p_1^{\sigma}}, \tag{7}$$

$$x_2 \equiv x_2(p_1,\, p_2,\, I) = \frac{I}{P p_2^{\sigma}}, \tag{8}$$

where $P \equiv (p_1^{1-\sigma} + p_2^{1-\sigma})^{\frac{1}{1-\sigma}}$ is the composite price index, giving the expenditure associated with one unit of total utility. Equations (7) and (8) show that each good's expenditure decreases with its own relative price with respect to the price index at a constant elasticity σ, and increases with income at a unit elasticity.

Firms. Each good is produced by a distinct firm. Firms 1 and 2 are heterogeneous in terms of production technology. In particular, let $c_i > 0$ be the constant marginal cost of firm $i = 1,\, 2$. We assume that the product market is monopolistically competitive. Each firm i solves the following maximization problem (by taking the composite price index, P as given):

$$\tilde{\pi}_i = \max_{p_i} (p_i - c_i) x_i(p_i,\, p_j,\, I) \tag{9}$$

for $i \neq j$. The first-order condition (Ramsey rule) is given by:

$$p_i \left(1 - \frac{1}{\varepsilon_i}\right) = c_i, \tag{10}$$

where ε_i is the price elasticity of good i. It follows from (7) and (8) that $\varepsilon_1 = \varepsilon_2 = \sigma$. Therefore, (10) implies

$$p_i = \frac{\sigma}{\sigma - 1} c_i \quad \text{for } i = 1,\, 2. \tag{11}$$

Because $\sigma > 1$, we have $p_i > c_i$, i.e., price of good i is a constant mark-up over its marginal cost of production. The Lerner index of each firm is given by $1/\sigma$, i.e., market power of the firms decreases with product substitutability.

Normalizing the low-skill wage, w_l to 1, and denoting by $w \equiv w_h/w_l$ the high-skill wage premium, lemma 3.1 summarizes firms' factor demands and cost functions.

Lemma 3.1. *The (conditional) factor demand and cost functions of firm $i = 1,\, 2$ are given by*

$$h_i(w,\, y_i) = \frac{1}{A} \left(\frac{\alpha}{w}\right)^{\zeta} z_i^{\zeta - 1} c_i^{\zeta} y_i, \tag{12}$$

$$l_i(w,\, y_i) = \frac{1}{A} (1 - \alpha)^{\zeta} c_i^{\zeta} y_i. \tag{13}$$

Firm i's cost function is linear in y_i, and the associated constant marginal cost is given by

$$c_i \equiv c(w,\, z_i) = \frac{1}{A} \left(\alpha^{\zeta} z_i^{\zeta - 1} w^{1-\zeta} + (1 - \alpha)^{\zeta}\right)^{\frac{1}{1-\zeta}} \tag{14}$$

with c_i increasing in w and decreasing in z_i.

Proof. In the appendix A.2. □

3.3 Determination of high-skill equilibrium premium and aggregate income

Next, we determine the high-skill wage premium, w and household income, I in the equilibrium of the second stage, that is, conditional on z_i for $i = 1, 2$. This equilibrium can be found by applying market clearing conditions in the factor markets:

$$L = l_1(w, y_1) + l_2(w, y_2), \tag{15}$$

$$H = h_1(w, y_2) + h_2(w, y_2), \tag{16}$$

and in the product markets:

$$y_1 = x_1(p_1,\, p_2,\, I), \tag{17}$$

$$y_2 = x_2(p_1,\, p_2,\, I). \tag{18}$$

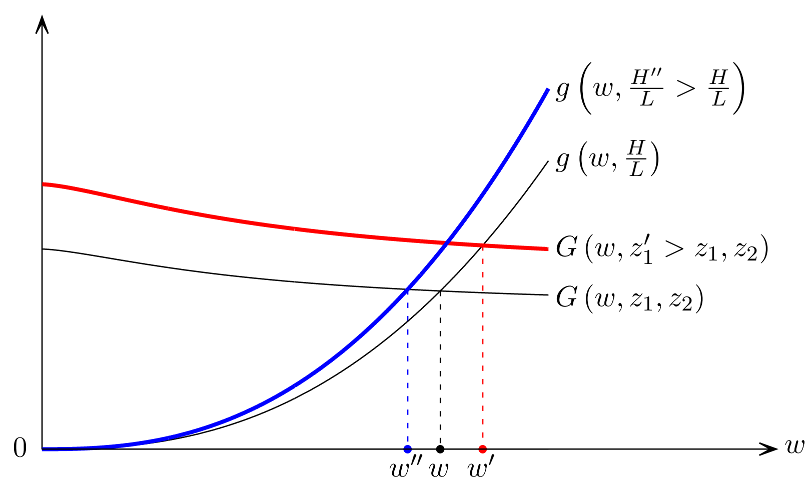
Equations (15)-(18) coupled with the firms' optimal price choice (11) and the definition of the marginal cost (14), yield a solution for all the endogenous variables in the economy, namely, prices $\{w, p_1, p_2\}$, quantities $\{y_1, y_2, l_1, l_2, h_1, h_2\}$, and income $\{I\}$. Moreover, one can show that this equilibrium is unique (see lemma A.2 in the appendix A.2).

Focusing on the high-skill wage premium, w, the next proposition characterizes its equilibrium interactions with variations in the SBTC and the relative supply of high-skill labor.

Proposition 3.1. *Given firm technologies $z_1 > z_2$ and $\sigma \geq \zeta$, the equilibrium high-skill wage premium w is increasing in z_i and decreasing in the relative supply of high-skill labor H/L.*

Proof. In the appendix A.2. □

Figure 3: Comparative statics of the equilibrium high-skill premium as a function of $(z_1,\, z_2, H/L)$.



Notes: This figure plots the functions $g(w, H/L)$ and $G(w, z_1, z_2)$ characterized in lemma A.2 for a parameterization where $z_1 > z_2$ and $\sigma > \zeta$. H/L refers to the relative supply of high-skill labor, while the z_i's refer to the SBTC of firms $i = 1, 2$. The lemma shows that the unique solution in w is given by $g(w, H/L) = G(w, z_1, z_2)$.

This result, depicted graphically in figure 3, generalizes the Tinbergen (1974) model for an environment with two goods and monopolistic competition. In fact, when the consumer does not value good 2, the equilibrium condition becomes

$$\left(\frac{1-\alpha}{\alpha}w\right)^{\zeta} \frac{H}{L} = z_1^{\zeta-1}.$$

The intuition for how w changes in equilibrium is the same in both environments. An improvement in the efficiency of the high-skill labor through an increase in z_i, implies excess demand in the market that is resolved with a higher price of that factor. A similar argument can be made for an increase in H/L that generates negative excess demand and therefore a lower w.

3.4 The equilibrium bidding for managers

In stage 1, firms bid for managers by posting managerial wages (r_1, r_2). Let m_i denote the managerial employment in firm $i = 1, 2$. Further, let

$$\pi_i(m_1, m_2) \equiv \tilde{\pi}_i(z(m_1), z(m_2))$$

be the profit of firm i when firms 1 and 2 employ m_1 and m_2 managers, respectively.[8] Importantly, the Hicks-neutral technological parameter A has no impact in the determination of the model equilibrium that is characterized in the following proposition.

Proposition 3.2. *For any equilibrium allocation of managers across the firms,* (m_1^*, m_2^*)*, the unique wage for managers is given by*

$$r_1 = r_2 = r(M) = \frac{\pi_1(M, 0) - \pi_1(0, M)}{M} = \frac{\pi_2(0, M) - \pi_2(M, 0)}{M}\,. \tag{19}$$

Moreover, $(m_1^*, m_2^*) = (M, 0)$ *or* $(m_1, m_2) = (0, M)$ *is an equilibrium managerial allocation across firms 1 and 2.*

Proof. In the appendix A.2. □

Given that managers enhance the efficiency of high-skill labor in the production function, the corresponding profit also changes. In particular, holding all else constant, a firm's profit increases with the number of managers employed. Firms compete for managers by biding a transfer that equals the profit opportunity cost of not securing managers. Since one manager not employed in one firm is one manager employed in the other firm, an equilibrium allocation ensures that all managers ultimately find employment in a leading firm. The total payment for managers, $r(M) \cdot M$, is then equal to the difference in profits, $\pi_i(M, 0) - \pi_i(0, M)$. This indicates that a firm's profit, net of the transfer to the managers, remains the same whether the firm employs their services or not since, in equilibrium, $\pi_i(M, 0) - r(M) \cdot M = \pi_i(0, M)$. It also means that the competition forces present in this environment make managers being residual claimants of the excess profit generated by their ability to increase the efficiency of high-skill labor.

We next show that the equilibrium managerial wage, $r(M)$ can be non-monotonic in managerial supply, M. This is driven by two countervailing forces— both the numerator of (19), $\pi_1(M, 0) - \pi_1(0, M)$ or $\pi_2(0, M) - \pi_2(M, 0)$, and the denominator increase with M.

[8] Analytical expressions for the profit functions $\pi_i(m_1, m_2)$ are derived in lemma A.3, appendix A.2.

Proposition 3.3. *There exist parameter values under which the equilibrium managerial wage is non-monotonic in managerial supply. In particular, for $\sigma = \zeta$,*

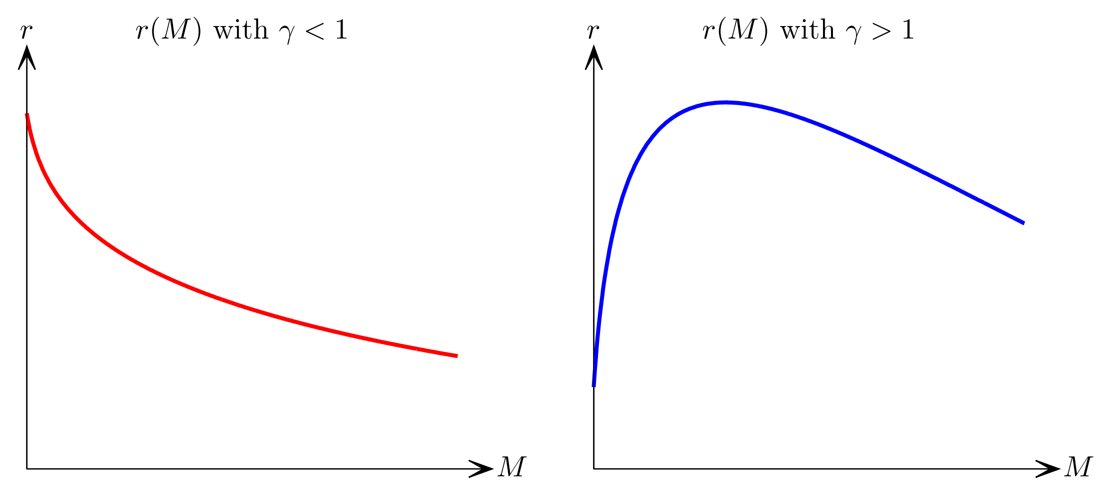
(a) *If $\gamma \leq 1$, then $r(M)$ is decreasing in M;*

(b) *If $\gamma > 1$, then $r(M)$ is either increasing in M or there is a unique $M^* > 0$ such that $r(M)$ is increasing (decreasing) in M according as $M \leq (>)\, M^*$.*

Proof. In the appendix A.2. $\square$

We recall, from equation (3), that the SBTC depends on γ through $z(m_i) \equiv z_0 + m_i^{\gamma}$. Proposition 3.3 demonstrates that when $\gamma \leq 1$, the managers' wage r decreases with their supply in the general equilibrium (left panel of figure 4). This result aligns with the implication in Proposition 3.1, which establishes a relationship between the supply of high-skill labor H and its compensatory wage w. Both results encapsulates the general intuition that prices reflect relative scarcity. However, this intuition is disrupted when $\gamma > 1$. In such case the relationship between the managers' wages and their supply exhibits a hump-shape, as displayed in the right panel of Figure 4. At a low level of managers in the economy, managers' wages actually increase with their supply, a trend that is reverted when the level of managers is high.

Figure 4: Manager's equilibrium wage with respect to supply



Notes: This figure plots the equilibrium managers' wage from proposition 3.2 for different values of the managers' supply in the economy. M refers to the total supply of managers in the economy and r refers to their equilibrium wage. Both lines are generated with a parameterization with $\sigma = \zeta = 2$, but the left panel uses a $\gamma < 1$ while the right panel uses a $2 > \gamma > 1$. Different parameterizations with $\sigma > \zeta$ generate similar patterns.

To better understand the mechanism behind the hump-shape result from proposition 3.3, we proceed by decomposing the impact of increasing the supply of managers on their wages into a direct and indirect effect. We isolate the direct effect of changing M on r, by evaluating equation (19) while keeping the high-skill wage constant at $w = \overline{w}$, which implies that the total household income is also constant at $I = \overline{I}$. To do this, we first note that the profit function can be written as

$$\tilde{\pi}_i^{direct} = \frac{1}{\sigma - 1} c_i y_i = \frac{c_i^{1-\sigma}}{c_1^{1-\sigma} + c_2^{1-\sigma}} \frac{\overline{I}}{\sigma}, \tag{20}$$

where the marginal cost is given by:

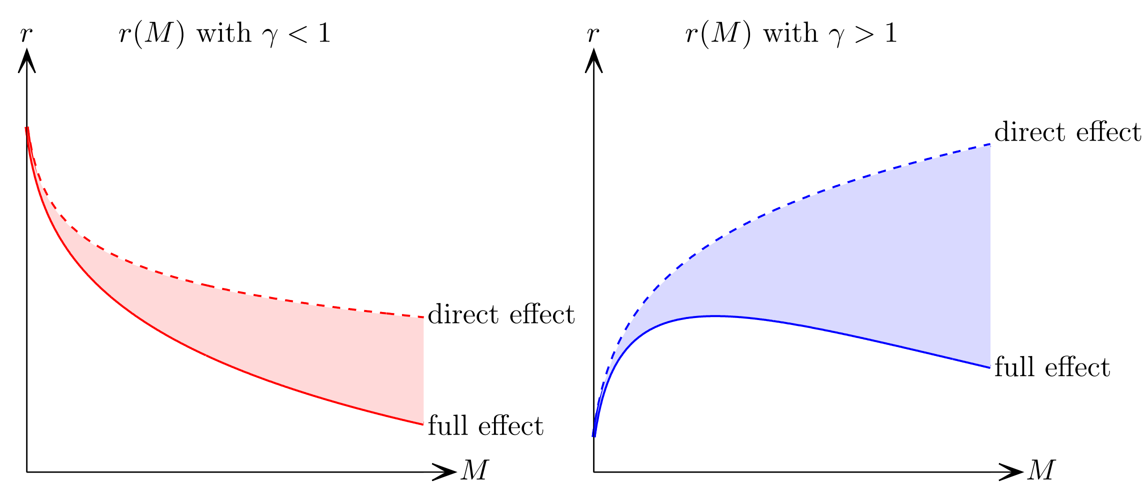
$$c_i = \frac{1}{A} \left(\alpha^{\zeta} z_i^{\zeta - 1} \overline{w}^{1-\zeta} + (1-\alpha)^{\zeta} \right)^{\frac{1}{1-\zeta}}. \tag{21}$$

The direct effect on the managers wage is then given by:

$$r^{direct} = \frac{\pi_i^{direct}(M, 0) - \pi_i^{direct}(0, M)}{M}. \tag{22}$$

The indirect effect is then just the difference between the full and the direct effect: $r^{indirect} = r - r^{direct}$. Figure 5 using equations (20)-(22), illustrates the application of this decomposition to the functions depicted in figure 4.

Figure 5: Decomposition of the manager's equilibrium wage with respect to supply



Notes: This figure decomposes the full effect of the manager's wage response to an increase in the managers supply described in figure 4. The dashed line isolates the direct effect using equation (22). The indirect effect is then just the difference between the full (the solid line) and the direct effect, highlighted with the shaded area. To compute the direct effect dashed lines, we fixed the high-skill wage w at some initial general equilibrium and let the supply of managers M grow.

The figure shows that the direct effect of increasing M on r always exceeds the full effect. An increase in z_i, induced by a rise in M, leads to a surge in firm i's profit due to a direct improvement in the production efficiency of the high-skill factor that also implies a reduced marginal cost. However, this simultaneously generates a higher demand for H, which is balanced in the market through an increased wage w. As the high-skill wage w also contributes positively to the marginal cost c_i, the decrease in marginal cost from an increase in z_i is less significant than it would be when considering only its direct effect. Hence, the negative indirect effect.

Moreover, proposition 3.3 proves that for a sufficiently large M, the indirect effect always overpowers the direct effect when $\gamma < 1$ and, for some cases, when $\gamma > 1$ (recall from equation (3) that $z_i = z(m_i) \equiv z_0 + m_i^\gamma$). In the context of the right panel of figure 5, when $\gamma > 1$, convexity of $z(m_i)$ generates a profit opportunity cost per unit of manager (as in equation (19) of proposition 3.3) that increases with the supply of managers, considering only its direct consequence. This effect dominates the full impact on r for low levels of M but, as M increases, the general equilibrium variation in prices ultimately sets a dominant indirect effect. The combined full effect creates the hump-shape depicted in the figure.

Table 3: Regression models for the manager wage premium

Dependent variable: log relative wage manager to high school	(I)	(II)
Regressors:		
relative employment of managers	2.50***	2.10***
	(0.35)	(0.55)
square of the relative employment of managers	-2.26***	-1.69*
	(0.77)	(0.97)
relative employment of college workers		0.023
		(0.026)
sectoral fixed effects	Yes	Yes
Observations	96	96
R-squared	0.68	0.69

Source: US Census samples and the American Community Survey.

We conclude this section by emphasizing that the different behavior of the manager's wage with its supply, as depicted in figure 4, offers relevant insights for parameterizing γ in order to align this model with real-world observations. Indeed, using the data outlined in section 2, we can project the managers wages against the supply of managers and the square of this variable to examine the evidence of a hump-shaped relationship. Table 3 presents the results of these regression. Both specifications (I) and (II) yield a negative and significant negative coefficient on the square of the managers' labor supply. This outcome provides suggestive evidence supporting the validity of the presented model when parameterized with $\gamma > 1$.

4 Quantitative Exercise

The preceding section unveiled a model underscoring the general equilibrium implications between the supply of managers and the wages of both managers and non-managers in the economy. We showed that an environment with a relative increase in the number of managers is consistent with a growing premium of both managers and high-skill workers. In this section, we present a simple extension of that model to allow for the quantification of the main innovation put forth in this paper. Specifically, we aim to address the question: how has the change in the supply of managers from 1950 to 2019 contributed to the evolution of the skill premium in the US economy?

From proposition 3.2, we know that an equilibrium is characterized by a corner solution where a single firm, which we can call the leading firm, hires all managers of the economy. We assume this is firm 1. The level of skill-biased technological change (SBTC), induced by managers in this leading firm, takes the form imposed by equation (3). Specifically, at time t

$$z_{1t} = z_0 + b \cdot m_t^{\gamma}, \tag{23}$$

where m_t represents the number of managers hired, and b is a time-invariant parameter that scales the effect of the manager in the production function but has otherwise no baring in the equilibrium definition used in proposition 3.2. Substituting equation (23) into the production function results in:

$$y_{1t} = A\left(\alpha S_t \left[\left(z_0 + b \cdot m_t^{\gamma}\right) h_{1t}\right]^{\frac{\zeta-1}{\zeta}} + (1-\alpha)\, l_{1t}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}}, \tag{24}$$

where h_{1t} and l_{1t} represent the amount of high-skill and low-skill workers hired, and S_t is an additional exogenous variable. We use S_t, common to both firms, as an additional free variable that allows the model to absorb all the variation in the equilibrium wages of the high-skill workers not explicitly accounted for in the current environment. That is, given a sequence of labor supplies $\{L_t, H_t, M_t\}$, one can always select a S_t that matches a particular value of w_t in the equilibrium. In this sense, S_t can be interpreted as a residual SBTC or a model wedge, as originally implemented in Tinbergen (1974) and in subsequent studies (Katz and Murphy, 1992; Card and DiNardo, 2002; or Acemoglu and Autor, 2011). In our quantification, we interpret S_t as encompassing all other forces that may affect the high-skill premium but are not included in the current model. These might relate to, but are not exclusively limited to capital-skill complementarities (Krusell et al., 2000), quality-adjusted high-skill labor supply (Carneiro and Lee, 2011), changes in the quality of goods consumed (Jaimovich et al., 2020), or structural changes of the economy (Buera et al., 2021).

One additional useful transformation of the production function implies re-writing equation (24) in terms of relative factor input variables. Let the total labor force of the economy at time t be given by the auxiliary variable $F_t \equiv L_t + H_t + M_t$. Then the production function becomes

$$y_{1t} = AF_t \left(\alpha S_t \left[\left(z_0 + b \cdot F_t^{\gamma} \tilde{m}_t^{\gamma} \right) \tilde{h}_{1t} \right]^{\frac{\zeta-1}{\zeta}} + (1-\alpha)\, \tilde{l}_{1t}^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}}, \tag{25}$$

where $\tilde{m}_t$, $\tilde{h}_{1t}$, and $\tilde{l}_{1t}$ are the input factors normalized to the total labor force in the economy. Provided with particular calibration of parameters $\{\sigma, \zeta, \alpha, A, z_0, b, \gamma\}$, the equilibrium definition described in section 3.1 allows for a structure where we can determine endogenously the premium of the high-skill workers w_t and the premium of the managers r_t using a set of relative aggregate input factors $\{\tilde{L}_t, \tilde{H}_t, \tilde{M}_t\}$, the size of the labor force F_t, and S_t as the residual SBTC. We now proceed to describe the calibration strategy of the model.

4.1 Calibration strategy

The parameters of the model are calibrated based on the aggregate data observations presented in figure 1 from section 2. Specifically, we utilize the observed sequence of labor shares in the US economy, $\{\tilde{L}_t, \tilde{H}_t, \tilde{M}_t\}$ for $t = 1950, 1960, ..., 2019$, along with an index capturing the size of the labor force, $\{F_t\}$, as model inputs. Given these inputs and a chosen set of parameters, the skill-biased technological change (SBTC) variable, S_t, is then adjusted to provide a model solution that generates an exact match with the sequence of data observations for the high-skill premium $\{w_t\}$. At this stage, we are left with an un-targeted model generated sequence of managers' premia $\{r_t^{model}\}$ which can be compared with the counterpart data-observed sequence $\{r_t^{data}\}$. The calibration is simply the outcome of choosing parameters that minimizes the mean-square distance between the model and data observations:

$$\Psi = (1/T) \sum_{t=1950}^{2019} \left[\log r_t^{model} - \log r_t^{data} \right]^2 \tag{26}$$

We incorporate additional discipline into the model by externally calibrating certain parameters. The elasticity of substitution between goods in the household utility function, as defined in equation (1), is fixed at $\sigma = 2.5$. This enables our model, which does not explicitly incorporate capital, to yield a labor share of 60%. For the elasticity of substitution between high-skill and low-skill labor in the production function (equation 25), we choose $\zeta = 2$. This is an intermediate value within the estimated range of 1.6 to 2.9 from Acemoglu and Autor (2011).[9] It is important to note that the parameter α is unidentified alongside S_t, since the latter is used to rationalize the observed sequence of w_t. For this reason, we use a simple normalization, setting $\alpha = 0.5$. Similarly, the common Hicks-neutral productivity does not influence the equilibrium outcome of either premium,

[9] Our choice is also close to the estimated elasticity of 1.8 in Ohanian et al. (2021), that revisits the estimation from Krusell et al. (2000) using 20 additional years of data. Different from our model, the production function in these papers allows for additional substitution between low-skill labor and capital.

so it is also normalized to $A = 1$.

One final externally set parameter is the constant z_0 from the manager-induced SBTC equation (23). Following Gabaix and Landier (2008) and Bao et al. (2022), we calibrate this parameter to reflect the share of manager wages in firms' total sales. Because we do not use data on firm total sales, we convert it info a share to wages with respect to value added. Another key difference is that while these studies focus on compensation of very top-level managers such as CEOs, our notion of managers is broader and includes mid-level management positions. Consequently, we target the aggregate managers' wage share to gross output at the beginning of our sample, which gives us a z_0 value of 2.5%.

Table 4: Model calibration

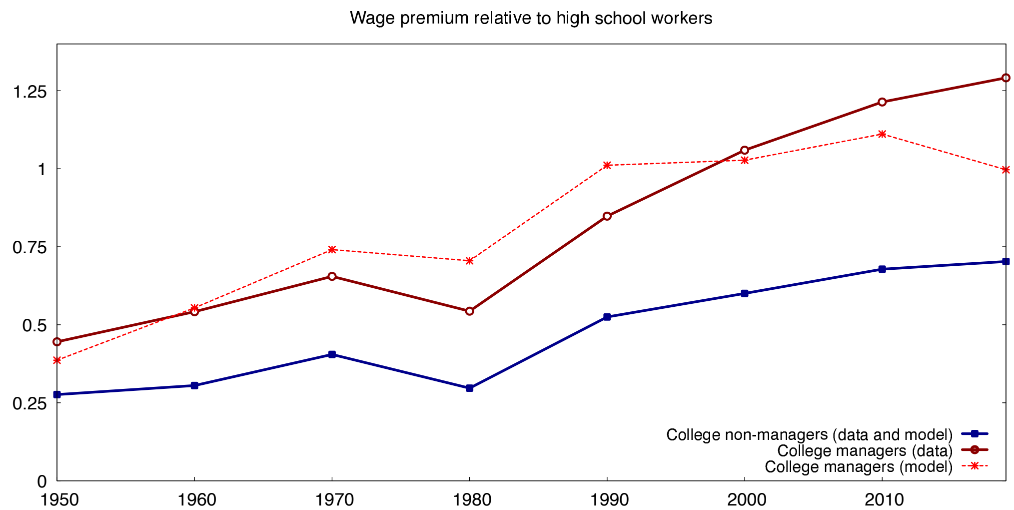
Parameter		Value	Target
Elast. of sub. between goods	σ	2.5	Economy-wide labor share of 60%
Elast. of sub. between high and low-skill labor	ζ	2.0	Acemoglu and Autor (2011)
Hicks-neutral productivity	A	1.0	Normalization
Production intensity of high-skill labor	α	0.5	Normalization
Size effect in SBTC induced by managers	z_0	0.025	Share of managers wages on sales
Importance of additional managers in SBTC	b	0.88	Minimum distance Ψ in (26)
Convexity of additional managers in SBTC	γ	1.04	Minimum distance Ψ in (26)

The remaining parameters b, γ are selected to minimize the distance criterion introduced in equation (26). Specifically, they are chosen to minimize the discrepancy between the manager's pay generated by the model and the corresponding data. Table 4 provides a summary of the calibration.

Figure 6 illustrates how well the model fits the data. It's important to note that the blue line in the figure, by design, represents both the data and the model evolution of the wage premium for high-skill labor, owing to the inclusion of the free SBTC variables S_t. However, the same cannot be said for the fit of the managers' premium, represented by the red line, where the fit is based on parameters that are held constant for the entire period. Despite this, the model successfully captures the general trend and level of managers' pay, generating a 39% premium in 1950 that peaks at 111% in 2010 and falls back to 100% in 2019.

Overall, the calibration strategy results in a mean square error across all data points of 7.3% (calculated as the square root of equation 26), corresponding to an average distance between the model predictions and data observations of 6.1% for the analyzed period. Despite providing a good fit, the manager's premium predicted in the model diverges from the data in the last observation of the time window. The calibration results and the internal mechanism of the model explained in proposition 3.3 explain this outcome. Specifically, our parameterization of $\gamma = 1.04$ is consistent with convexity in the managers induced SBTC equation (23). Furthermore, the data show an uninterrupted increase in the relative supply of managers in the labor force. Combined, these two factors generate a model environment in which the premium for managers declines due to general equilibrium effects when their supply in the economy becomes sufficiently large. Our calibration suggests that tipping point occurred at around 2010.

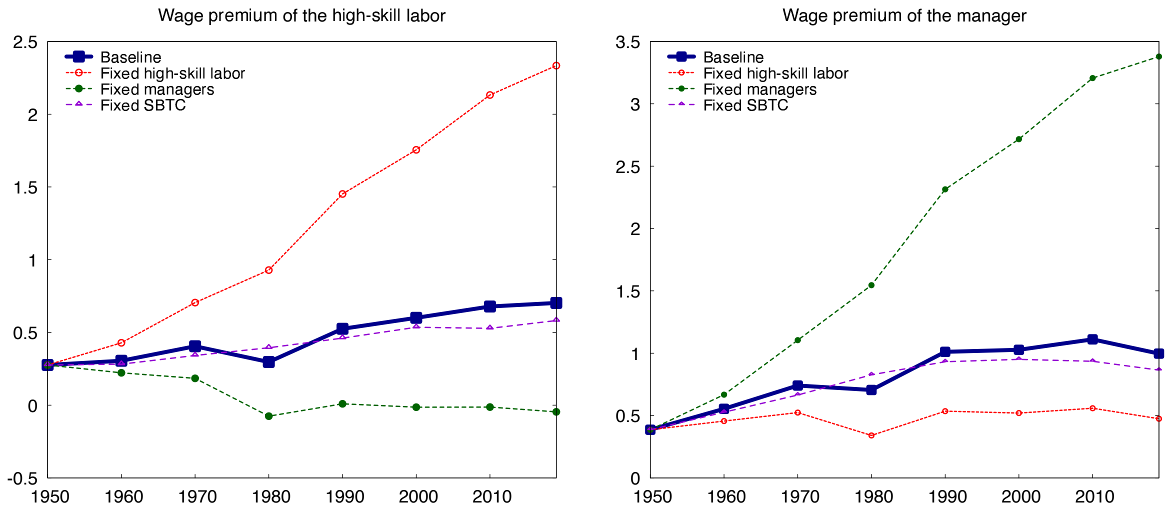
Figure 6: Model fit of managers premium: data vs. model



4.2 Decomposing the expansion in the high-skill labor and managers' premium

Drawing on the results generated by the calibration delineated in table 4, we can employ the model to dissect the sources of wage inequality. This is accomplished by running counterfactual scenarios where certain variables are kept at a constant level throughout the analysis period. Specifically, we investigate how the wage premium would alter if either the relative supply of managers had remained constant at 1950s levels, or if the relative supply of high-skill labor had remained static. These counterfactuals are conducted while keeping the same calibration and sequence of exogenous SBTC, $\{S_t\}$, as implied in figure 6. For further comparability, we also run a counterfactual where we let the relative supply of managers and high-skill workers to evolve as observed in the data, but the exogenous SBTC is kept at 1950s levels. Figure 7 shows the implied wage premiums of these counterfactuals and table 5 summarizes the results.

Figure 7: Model counterfactuals under fixed high-skill labor, fixed managers, and fixed SBTC



The figure underscores the role that the shifting composition of the labor force plays in wage premiums. Fixing the share of high-skill labor at its 1950s levels, while maintaining the influence of the sequence of exogenous SBTC, changes the balance of supply and demand forces towards the later, a result that is common in the skill-premium literature. The increase in efficiency induced by the SBTC generates higher demand for high-skill labor that is resolved in the market with higher wages (red line in the left panel of the figure). Interestingly, the increase of the managers premium (depicted in the red line at the right panel of the figure) remains comparatively subdued, with an uptick of only 8 percentage points. This outcome stems from the fact that, in the model, most of the benefit of hiring managers are realized through a more intense use of high-skill labor. However, the increase in a firm's profit from using additional high-skill labor is relatively small due to the high wages of high-skill workers.

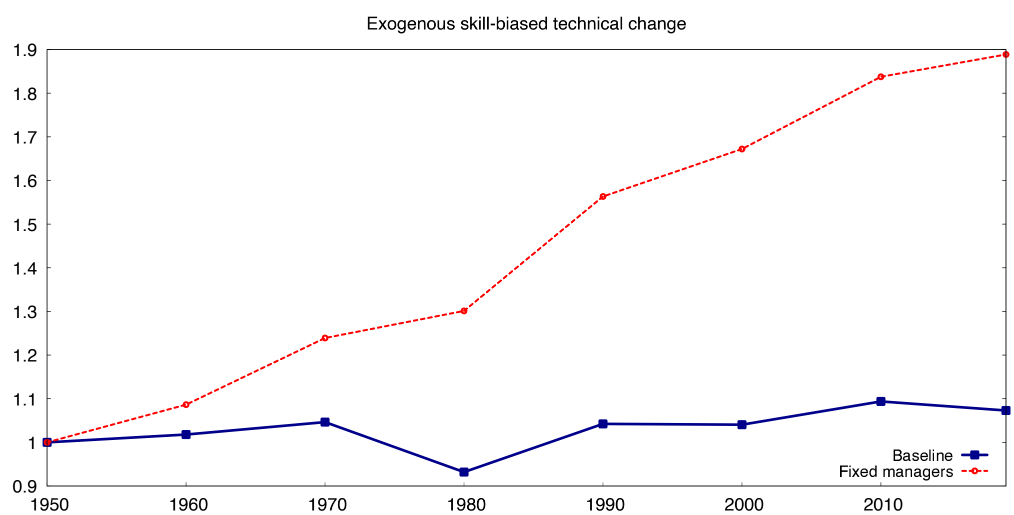
Keeping the share of managers constant in the model (the green lines in the figure) provides a mirror image of this result. We recall that an increase in the share of managers in the economy improves the production efficiency of high-skill workers through equation (23). Therefore, with a constant share of managers in the economy, demand for high-skill labor due to higher efficiency does not increase as much as it would otherwise. However, the marginal impact of an additional manager in a firm's profit increase due to suppression of high-skill wages. This translates into a larger premium of managers in the economy.

Table 5: Change in wage premium between 1950-2019 in the model counterfactuals

	change of the premium 1950-2019	
	for high-skill workers	for managers
Scenario:		
Baseline	42pp	61pp
Under fixed high-skill labor	205pp	8pp
Under fixed managerial labor	-32pp	299pp
Under fixed SBTC	30pp	47pp

Additionally, a comparison between the blue and purple lines in the figure shows that fixing the exogenous SBTC to its 1950s levels does not substantially alter the dynamics of the high-skill or managerial premium over the period. In particular, the increase in the high-skill premium decreases from 42 to 30 percentage points, while the increase in the managers' premium decreases from 61 to 47 percentage points. This suggests that the inclusion of managers in our model provides a mechanism that largely mitigates the reliance of an exogenous model wedge, or in our case, the SBTC, to match the evolution of the skill-premium of high-skill workers.

Figure 8: Skill-biased technical change required to match the high-skill premium in the data



An alternative way of uncovering this result involves in recalculating the required SBTC wedge that rationalizes the observed path in the high-skill wages when the share of managers in the economy is held constant. Figure 8 contrasts the evolution of the SBTC under the baseline calibration with this counterfactual scenario. Notably, under the baseline scenario where the share of managers varies in line with the data, the necessary increase in the SBTC that matches the evolution of the high-skill premium is a mere 7%. Instead, when the increase in the managers share is shutdown, the required change in the SBTC escalates to 88% over the period. This substantial difference underscores that accounting for the relationship between production, availability of managers, and firm competition for their services can provide an important channel that explains the skill-premium puzzle evident in the data.

5 Conclusion

This paper explores how the availability and competition for managerial services contributes to the rise of the skill premium in the US economy. An environment where firms hold market power and managers enhance production efficiency leads to competition for the services of managers where their compensation is associated with the profit opportunity cost of not hiring managers. When the effect of managers on firms' production is sufficiently convex, this leads to a compensation of managers that is hump-shaped, increasing when the supply of managers is scarce and decreasing when it becomes abundant. At the same time, an increase of the share of managers in the labor force generates an increase in demand for the high-skill labor if the managerial services provided increase disproportionally the relative efficiency of high-skill labor instead of low-skill labor. This effect contributes further to an increase in the high-skill premium.

A full characterization of a model that highlights how these relationships can emerge as economic equilibrium outcomes is presented. The model extends the canonical Tinbergen (1974) environment by explicitly incorporating firm competition under market power. By enabling managers to influence the production efficiency as in Gabaix and Landier (2008), we show that a Nash equilibrium exists where firms use profits to bid for managerial services. Furthermore, the characterization of the equilibrium reveals that, under some parameterizations, the model generates outcomes that are consistent with observations on the high-skill and managerial premium for the US economy in the past 70 years. Using census ACS data, we document a concurrent increase in both the relative supply of managers (from 4% to 20%) and high-skill workers (from 12% to 33%). During the same period, the high-skill wage premium surged by 42 percentage points, while the managers premium rose 61 percentage points.

Rather than categorizing these data patterns as puzzling, a reasonable calibration of the model can account for these observed dynamics. In our framework, the increase in the relative supply of managers can almost entirely explain the high-skill premium observed in the data, thereby eliminating the need to rely on exogenous model wedges such as skill-biased technical change.

It is important to note that the results were derived using a stylized model that was left purposely simple to highlight the mechanisms at play. Notably, we have disregarded other forces that may also contribute to the increase in the high-skill premium, such as, capital-skill complementarities, quality-adjusted high-skill labor supply, changes in the quality of goods consumed, or structural changes of the economy.[10] Nevertheless, one advantage of using a simple model lies its capacity to incorporate additional features. Model extensions along these lines and deeper data analyses are left for future research.

[10]These forces have been studied in Krusell et al. (2000); Carneiro and Lee (2011); Jaimovich et al. (2020); Buera et al. (2021).

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A Appendix

A.1 Additional details of the empirical section

TO ADD LATER.

A.2 Proofs of lemmas and propositions

The claims made here are all associated with the equations and definitions of section 3. We start with a simple lemma (A.1) that just re-writes product demands and profits as functions of marginal costs and total income.

Lemma A.1. *Given the constant marginal costs of the two firms,* (c_1, c_2)*, the optimal consumption of each good, and the profit of each firm are given by*

$$x_1(c_1, c_2) = \frac{(\sigma-1)I}{\sigma c_1^{\sigma}(c_1^{1-\sigma}+c_2^{1-\sigma})}, \quad x_2(c_1, c_2) = \frac{(\sigma-1)I}{\sigma c_2^{\sigma}(c_1^{1-\sigma}+c_2^{1-\sigma})}, \tag{27}$$

$$\tilde{\pi}_1(c_1, c_2) = \frac{I}{\sigma}\cdot\frac{c_1^{1-\sigma}}{c_1^{1-\sigma}+c_2^{1-\sigma}}, \quad \tilde{\pi}_2(c_1, c_2) = \frac{I}{\sigma}\cdot\frac{c_2^{1-\sigma}}{c_1^{1-\sigma}+c_2^{1-\sigma}}. \tag{28}$$

Proof. Using (10), the composite is given by

$$P(c_1, c_2) = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left(c_1^{1-\sigma}+c_2^{1-\sigma}\right).$$

Substituting the above and (10) into (7) and (8), we the expressions in (27). Firm i's profit is given by

$$\tilde{\pi}_i(c_1, c_2) = (p_i - c_i)x_i(c_1, c_2) = \left(\frac{\sigma c_i}{\sigma-1} - c_i\right)x_i(c_1, c_2).$$

Substituting $x_i(c_1, c_2)$ for $i = 1, 2$, we obtain the expressions in (28). □

The next lemma (3.1) uses the firms' production functions to determine factor demand functions and marginal costs.

Lemma 3.1. *The (conditional) factor demand and cost functions of firm* $i = 1, 2$ *are given by*

$$h_i(w, y_i) = \frac{1}{A}\left(\frac{\alpha}{w}\right)^{\zeta} z_i^{\zeta-1} c_i^{\zeta} y_i, \tag{12}$$

$$l_i(w, y_i) = \frac{1}{A}(1-\alpha)^{\zeta} c_i^{\zeta} y_i. \tag{13}$$

Firm i*'s cost function is linear in* y_i*, and the associated constant marginal cost is given by*

$$c_i \equiv c(w, z_i) = \frac{1}{A}\left(\alpha^{\zeta} z_i^{\zeta-1} w^{1-\zeta} + (1-\alpha)^{\zeta}\right)^{\frac{1}{1-\zeta}} \tag{14}$$

with c_i increasing in w and decreasing in z_i.

Proof. The first-order conditions with respect to h_i and l_i are respectively given by:

$$
\begin{aligned}
A\alpha z_i^{\frac{\zeta-1}{\zeta}} h_i^{-\frac{1}{\zeta}} \left(\alpha(z_i h_i)^{\frac{\zeta-1}{\zeta}} + (1-\alpha) l_i^{\frac{\zeta-1}{\zeta}}\right)^{\frac{1}{\zeta-1}} &= w, \\
A(1-\alpha) l_i^{-\frac{1}{\zeta}} \left(\alpha(z_i h_i)^{\frac{\zeta-1}{\zeta}} + (1-\alpha) l_i^{\frac{\zeta-1}{\zeta}}\right)^{\frac{1}{\zeta-1}} &= 1.
\end{aligned}
$$

The above conditions imply

$$
l_i = \left(\frac{(1-\alpha)w}{\alpha}\right)^{\zeta} z^{1-\zeta} h_i
$$

Substituting the above into the production function of firm i, we obtain (12). The steps to obtain (13) are similar. The cost function of firm i is given by

$$
C_i(w,\, z_i,\, y_i) = [w h_i(w,\, y_i) + l_i(w,\, y_i)] y_i = \underbrace{\frac{1}{A}\left(\alpha^{\zeta} z_i^{\zeta-1} w^{1-\zeta} + (1-\alpha)^{\zeta}\right)^{\frac{1}{1-\zeta}}}_{c(w,\, z_i)} \cdot y_i.
$$

This completes the proof of the lemma. ☐

The unique determination of the second stage equilibrium, taking z_1 and z_2 as given, is characterized in lemma A.2 by solving the model for the relative price of the high-skill labor w.

Lemma A.2. *Given firm technologies, $(z_1,\, z_2)$ and $\sigma \geq \zeta$, the equilibrium high-skill premium, denoted by $w(z_1,\, z_2)$, is uniquely determined by*

$$
G(w,\, z_1,\, z_2) = g(w), \tag{29}
$$

where

$$
\begin{aligned}
G(w,\, z_1,\, z_2) &\equiv \frac{z_1^{\zeta-1}\left(\alpha^{\zeta} z_1^{\zeta-1} w^{1-\zeta} + (1-\alpha)^{\zeta}\right)^{\frac{\zeta-\sigma}{1-\zeta}} + z_2^{\zeta-1}\left(\alpha^{\zeta} z_2^{\zeta-1} w^{1-\zeta} + (1-\alpha)^{\zeta}\right)^{\frac{\zeta-\sigma}{1-\zeta}}}{\left(\alpha^{\zeta} z_1^{\zeta-1} w^{1-\zeta} + (1-\alpha)^{\zeta}\right)^{\frac{\zeta-\sigma}{1-\zeta}} + \left(\alpha^{\zeta} z_2^{\zeta-1} w^{1-\zeta} + (1-\alpha)^{\zeta}\right)^{\frac{\zeta-\sigma}{1-\zeta}}}, \\
g(w) &\equiv \frac{H}{L}\left(\frac{(1-\alpha)w}{\alpha}\right)^{\zeta}.
\end{aligned}
$$

If $z_i > z_j$ for $i,\, j = 1,\, 2$ and $i \neq j$, the equilibrium high-skill premium, $w(z_i,\, z_j)$ is a increasing in z_i. The aggregate income, $I(z_1,\, z_2)$ is determined by

$$
I = \frac{\sigma L}{(\sigma-1)(1-\alpha)^{\zeta}} \cdot \frac{\left(\alpha^{\zeta} z_1^{\zeta-1} w^{1-\zeta} + (1-\alpha)^{\zeta}\right)^{\frac{1-\sigma}{1-\zeta}} + \left(\alpha^{\zeta} z_2^{\zeta-1} w^{1-\zeta} + (1-\alpha)^{\zeta}\right)^{\frac{1-\sigma}{1-\zeta}}}{\left(\alpha^{\zeta} z_1^{\zeta-1} w^{1-\zeta} + (1-\alpha)^{\zeta}\right)^{\frac{\zeta-\sigma}{1-\zeta}} + \left(\alpha^{\zeta} z_2^{\zeta-1} w^{1-\zeta} + (1-\alpha)^{\zeta}\right)^{\frac{\zeta-\sigma}{1-\zeta}}}.
$$

Proof. Using $y_i = x_i(c_1, c_2)$, it follow from (12) and (13) that

$$L = l_1(w, y_1) + l_2(w, y_2) = A^{\zeta-1}(1-\alpha)^{\zeta}\left(c_1^{\zeta}x_1(c_1, c_2) + c_2^{\zeta}x_2(c_1, c_2)\right), \tag{30}$$

$$H = h_1(w, y_1) + h_2(w, y_2) = A^{\zeta-1}(\alpha/w)^{\zeta}\left(z_1^{\zeta-1}c_1^{\zeta}x_1(c_1, c_2) + z_2^{\zeta-1}c_2^{\zeta}x_2(c_1, c_2)\right). \tag{31}$$

Substitute $c_i = c(w, z_i)$ as in (14) into the expressions of $x_i(c_1, c_2)$ for $i = 1, 2$ in order to express the consumptions of the two goods as functions of (w, z_1, z_2). Then, divide (30) by (31) to get the equilibrium skill-premium equation in (29). Note that

$$\frac{\partial G}{\partial w} = -\frac{(\sigma-\zeta)(\alpha(1-\alpha))^{\zeta}(\sigma-1)\left[\left(\alpha^{\zeta}z_1^{\zeta-1}w^{1-\zeta}+(1-\alpha)^{\zeta}\right)\left(\alpha^{\zeta}z_1^{\zeta-2}w^{1-\zeta}+(1-\alpha)^{\zeta}\right)\right]^{\frac{\sigma-\zeta}{\zeta-1}-1}}{w^{\zeta}\left(z_1^{\zeta-1}-z_2^{\zeta-1}\right)^{-2}\left[\left(\alpha^{\zeta}z_1^{\zeta-1}w^{1-\zeta}+(1-\alpha)^{\zeta}\right)^{\frac{\sigma-\zeta}{\zeta-1}}+\left(\alpha^{\zeta}z_2^{\zeta-1}w^{1-\zeta}+(1-\alpha)^{\zeta}\right)^{\frac{\sigma-\zeta}{\zeta-1}}\right]^{2}}.$$

Thus, $G(w, z_1, z_2)$ is decreasing in w because, by assumption, $\sigma \geq \zeta$.[11] Next, $\lim_{w\to\infty} G(w, z_1, z_2) = \frac{1}{2}\left(z_1^{\zeta-1} + z_2^{\zeta-1}\right) \in (0, \infty)$, and because G decreases with w, we have $G(0, z_1, z_2) > 0$. On the other hand, $g(w)$ is a strictly increasing and convex function with $g(0) = 0$ and $\lim_{w\to\infty} g(w) \to \infty$. Thus, by the Intermediate value theorem, the intersection between $G(w, z_1, z_2)$ and $g(w)$ is unique, which defines a unique $w(z_1, z_2)$.

Let $a(w, z_i) \equiv \left(\alpha^{\zeta}z_i^{\zeta-1}w^{1-\zeta} + (1-\alpha)^{\zeta}\right)^{\frac{\sigma-\zeta}{\zeta-1}}$. It is immediate to show that $\partial a(w, z_i)/\partial z_i > 0$. Note that we can write

$$G(w, z_i, z_j) \equiv z_j^{\zeta-1} + \frac{a(w, z_i)}{a(w, z_i) + a(w, z_j)}\left(z_i^{\zeta-1} - z_j^{\zeta-1}\right).$$

From the above it follows that $G(w, z_i, z_j)$ is increasing in z_i if $z_i > z_j$. Because $g(w)$ is an increasing function of w, which does not depend on z_1 and z_2, $w(z_i, z_j)$ is increasing in z_i. The first part of the Lemma is described in Figure 3.

Finally, note that

$$I = p_1x_1(c_1, c_2) + p_2x_2(c_1, c_2) = \frac{\sigma}{\sigma-1}\{c_1x_1(c_1, c_2) + c_2x_2(c_1, c_2)\}$$

which implies the expression of aggregate household income in the lemma. □

With the results from lemma A.2 one can further characterize some comparative statics impli-

[11]When $\sigma < \zeta$, $G(w, z_1, z_2)$ is increasing in w. Because

$$\lim_{w\to 0} G(w, z_1, z_2) = \frac{z_1^{\sigma-1} + z_2^{\sigma-1}}{z_1^{\sigma-\zeta} + z_2^{\sigma-\zeta}} > 0$$

for any (σ, ζ) and $\lim_{w\to\infty} G(w, z_1, z_2) < \infty$, the equilibrium $w(z_1, z_2)$ exists. However, the unicity cannot be guaranteed. If there are multiple equilibrium premia, choose the highest one, and our results in Lemma A.2 hold.

cations on the high-skill wage premium w. This is shown in proposition 3.1.

Proposition 3.1. *Given firm technologies $z_1 > z_2$ and $\sigma \geq \zeta$, the equilibrium high-skill wage premium w is increasing in z_i and decreasing in the relative supply of high-skill labor H/L.*

Proof. From lemma A.2, we have that the equilibrium w is the unique solution of:

$$G(w,\, z_1,\, z_2) = g(w, H/L). \tag{32}$$

The lemma also shows that $\partial G/\partial w < 0$, $\partial g/\partial w > 0$, $\partial g/\partial(H/L) > 0$, and $\partial G/\partial z_1 > 0$. Then, applying the implicit function theorem gives:

$$\begin{aligned} \frac{dw}{d\,(H/L)} &= \frac{\partial g/\partial\,(H/L)}{\partial G/\partial w - \partial g/\partial w} < 0, \\ \frac{dw}{dz_1} &= -\frac{\partial G/\partial z_1}{\partial G/\partial w - \partial g/\partial w} > 0. \end{aligned}$$

□

Finally, in the following lemma we describe the profits of the firms in the equilibrium of the second stage.

Lemma A.3. *Given firm technologies, $(z_1,\, z_2)$, the equilibrium profit of firm $i = 1,\, 2$ is*

$$\tilde{\pi}_i(z_1,\, z_2) = \frac{L(1-\alpha)^{-\zeta}}{\sigma - 1} \cdot \frac{\left(\alpha^\zeta z_i^{\zeta-1} w^{1-\zeta} + (1-\alpha)^\zeta\right)^{\frac{1-\sigma}{1-\zeta}}}{\left(\alpha^\zeta z_1^{\zeta-1} w^{1-\zeta} + (1-\alpha)^\zeta\right)^{\frac{\zeta-\sigma}{1-\zeta}} + \left(\alpha^\zeta z_2^{\zeta-1} w^{1-\zeta} + (1-\alpha)^\zeta\right)^{\frac{\zeta-\sigma}{1-\zeta}}}\,.$$

Proof. Immediately follows from Lemmas A.1 and A.2. □

With the results on the stage 2 of the equilibrium, that is, conditional on z_1 and z_2, we move now to the equilibrium allocation of managers across firms. The next proposition determines the payment for managers in the economy and an allocation between firm 1 and 2 resulting from the bidding process defined in section 3.

Proposition 3.2. *For any equilibrium allocation of managers across the firms, $(m_1^*,\, m_2^*)$, the unique wage for managers is given by*

$$r_1 = r_2 = r(M) = \frac{\pi_1(M,\, 0) - \pi_1(0,\, M)}{M} = \frac{\pi_2(0,\, M) - \pi_2(M,\, 0)}{M}\,. \tag{19}$$

Moreover, $(m_1^*,\, m_2^*) = (M,\, 0)$ or $(m_1,\, m_2) = (0,\, M)$ is an equilibrium managerial allocation across firms 1 and 2.

Proof. Let (m_1, m_2) be the allocation of managers in firms 1 and 2, and M be the aggregate supply of managers so that $m_1 \equiv m$ and $m_2 \equiv M - m$. Further let r_i be the wage for each manager offered by firm $i = 1, 2$. In a Nash equilibrium we have

$$\pi_1(m, M - m) - r_1 \cdot m \geq \pi_1(0, M), \tag{33}$$

$$\pi_1(m, M - m) - r_1 \cdot m \geq \pi_1(M, 0) - r_1 \cdot M. \tag{34}$$

The first inequality asserts that firm 1 has higher net profit by employing $m \in (0, M)$ managers rather than employing no managers. The second inequality, on the other hand, asserts that employing all M managers is less profitable for firms 1 than hiring $m \in (0, M)$ managers. Similarly, for firm 2 we have

$$\pi_2(m, M - m) - r_2 \cdot (M - m) \geq \pi_2(M, 0), \tag{35}$$

$$\pi_2(m, M - m) - r_2 \cdot (M - m) \geq \pi_2(0, M) - r_2 \cdot M. \tag{36}$$

We first show that, in equilibrium, $r_1 = r_2 = r$. Suppose on the contrary that $r_1 < r$. In this case, firm 2 gets all the M managers, and hence, firm 1's deviation profit is given by $\pi(0, M)$. However, from (33) it follows that $\pi(0, M) \leq \pi(m, M - m) - r \cdot m$. Thus, $r_1 < r = r_2$ is not a profitable deviation. Next, consider a deviation $r_1 > r$ by firm 1. In this case, firm 1's profit becomes $\pi_1(M, 0) - r_1 \cdot M$ as this firm gets all the M managers. Note that $\pi_1(M, 0) - r_1 \cdot M \leq \pi_1(m, M - m) - r_1 \cdot m < \pi_1(m, M - m) - r \cdot m$. The first inequality follows from (34), and $r_1 > r$ implies the second inequality. Therefore, $r_1 > r = r_2$ is not a profitable deviation for firm 1. Similar argument goes for firm 2. Therefore, in equilibrium, we have $r_1 = r_2 = r$.

Next, note that $\pi_1(m_1, m_2) + \pi_2(m_1, m_2) = L(1 - \alpha)^{-\zeta}/(\sigma - 1)$. Thus, (35) and (36) can respectively written as

$$r_2 \cdot (M - m) \leq \pi_1(M, 0) - \pi_1(m, M - m), \tag{37}$$

$$r_2 \cdot m \geq \pi_1(m, M - m) - \pi_1(0, M). \tag{38}$$

At $r_1 = r_2 = r$, from (33) and (38) it follows that

$$r \cdot m = \pi_1(m, M - m) - \pi_1(0, M). \tag{39}$$

On the other hand, (34) and (37) together imply that

$$r \cdot (M - m) = \pi_1(M, 0) - \pi_1(m, M - m). \tag{40}$$

Adding (39) and (40), we obtain

$$r = \frac{\pi_1(M,\,0) - \pi_1(0,\,M)}{M} \equiv r(M).$$

The second equality in (19) follows from the fact that $\pi_1(M,\,0)+\pi_2(M,\,0) = \pi_2(0,\,M)+\pi_2(0,\,M) = L(1-\alpha)^{-\zeta}/(\sigma-1)$. To show that $(M,\,0)$ or $(0,\,M)$ is an equilibrium allocation, note that both $m^* = M$ and $m^* = 0$ solve (39) and (40), the two equilibrium conditions at the bidding stage. $\square$

sdfasfasfsaf

Proposition 3.3. *There exist parameter values under which the equilibrium managerial wage is non-monotonic in managerial supply. In particular, for $\sigma = \zeta$,*

(a) *If $\gamma \leq 1$, then $r(M)$ is decreasing in M;*

(b) *If $\gamma > 1$, then $r(M)$ is either increasing in M or there is a unique $M^* > 0$ such that $r(M)$ is increasing (decreasing) in M according as $M \leq (>)\, M^*$.*

Proof. Let us consider $(m_1^*,\, m_2^*) = (M,\, 0)$. The proof is the same for $(m_1^*,\, m_2^*) = (0,\, M)$ because of symmetry. Note that, at managerial allocation, $(M,\,0)$, $z_1 = z_0 + M^\gamma$ and $z_2 = z_0$. Note that z_1 is strictly increasing in M with $\lim_{M\to 0} z_1 = z_0$ and $\lim_{M\to\infty} z_1 = \infty$. Using Lemma A.3, the equilibrium managerial wage can be written as a function of z_1:

$$r(z_1) = B \cdot \frac{z_1^{\sigma-1} - z_0^{\sigma-1}}{(z_1 - z_2)^{\frac{1}{\gamma}} \left(z_1^{\sigma-1} + z_0^{\sigma-1}\right)^{\frac{\sigma-1}{\sigma}}}, \tag{41}$$

where

$$B \equiv \frac{L}{2(\sigma-1)} \cdot \frac{\alpha}{1-\alpha} \left(\frac{2H}{L}\right)^{\frac{\sigma-1}{\sigma}}.$$

Note that

$$r'(z_1) = \underbrace{\frac{B(z_1 - z_0)^{-\frac{\gamma+1}{\gamma}} \left(z_1^{\sigma-1} + z_0^{\sigma-1}\right)^{\frac{1}{\sigma}-2}}{\gamma\sigma}}_{>0} \cdot H(z_1),$$

where

$$H(z_1) \equiv \sigma z_0^{2(\sigma-1)} + \gamma(\sigma-1)(2\sigma-1) z_0^{\sigma-1} z_1^{\sigma-2}(z_1 - z_0) + z_1^{2\sigma-3}(\gamma(\sigma-1)(z_1 - z_0) - \sigma z_0).$$

Therefore,

$$\operatorname{sign}[r'(z_1)] = \operatorname{sign}[H(z_1)].$$

Observe that $H(z_0) = 0$. Note that

$$H'(z_1) = z_1^{\sigma-3} \cdot h(z_1),$$

where

$$h(z_1) \equiv \gamma(\sigma-1)(2\sigma-1)z_0^{\sigma-1}((\sigma-1)(z_1-z_0)+z_0)+(\sigma-1)z_1^{\sigma-1}[\gamma(2(\sigma-1)(z_1-z_0)+z_0)-2\sigma z_1].$$

Thus,

$$\text{sign}[H'(z_1)] = \text{sign}[h(z_1)].$$

Note that $h(z_0) = 2(\gamma-1)(\sigma-1)\sigma z_0^{\sigma}$, and hence, $h(z_0) > (\leq)\, 0$ according as $\gamma > (\leq)\, 1$. On the other hand, $\lim_{z_1\to\infty} h(z_1) = -\infty$. First, we analyze the case when $\gamma \leq 1$. It is easy to show that $h'(z_1) < 0$ whenever $\gamma \leq 1$. Because $h(z_0) \leq 1$ for $\gamma \leq 1$, we have $h(z_1) \leq 0$ for all z_1. This implies $H'(z_1) < 0$ with $H(z_0) = 0$, and hence, $H(z_1) \leq 0$ or equivalently, $r'(z_1) \leq 0$. This completes the proof of part (a).

To show part (b), consider $\gamma > 1$. In this case, $h(z_0) > 0$, and $\lim_{z_1\to\infty} h(z_1) = -\infty$. Therefore, by Intermediate Value theorem, there is a $\bar{z}_1 > z_0$ such that $h(\bar{z}_1) = 0$. It is easy to show that $\bar{z}_1$ is unique.[12] Thus, $h(z_1) > (<)\, 0 \iff H'(z_1) > (<)\, 0$ according as $z_1 < (>) \bar{z}_1$. Therefore, $H'(z_1) > (<)\, 0$ according as $z_1 < (>) \bar{z}_1$. Because $H(z_0) = 0$, there are two possibilities—(i) although $H(z_1)$ is a decreasing function beyond $\bar{z}_1$, it may remain positive for all z_1, and hence, $r'(z_1) > 0$; (ii) $H(z_1)$ crosses the horizontal axis at a $z_1^* > \bar{z}_1$, and eventually becomes negative in, which case $r(z_1)$ is hump-shaped, attaining the maximum at $z_1 = z_1^*$ (see Figure 4).

The result follows from the fact that there is one-to-one correspondence between M and z_1, and letting $M^* = (z_1^* - z_0)^{\frac{1}{\gamma}}$. □

A.3 Additional results of the quantitative exercise

TO ADD LATER.

[12] If $\gamma \leq \frac{\sigma^2}{(\sigma-1)(\sigma+1)}$, then $h'(z_1) < 0$, and hence, the intersection is unique. On the other hand, $\gamma > \frac{\sigma^2}{(\sigma-1)(\sigma+1)}$, then $h'(z_1) > 0$ for small z_1, but $h'(z_1) < 0$ for large z_1. In this case also, the intersection is unique.